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ESSAYS ON REAL OPTIONS

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This thesis is dedicated to my parents.

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Sommario

La tesi esplora l'uso del metodo delle opzioni reali nella risoluzione di vari problemi. Obiettivo del lavoro è sviluppare una metodologia per la valutazione delle opzioni reali, che consenta l'analisi, la pianificazione e la gestione degli investimenti realizzati in condizioni di elevata incertezza. Oggetto della ricerca sono i progetti di investimento delle imprese, realizzati in condizioni di incertezza e in differenti condizioni di mercato. La ricerca è incentrata nella determinazione del valore delle opzioni di azioni alternative e l'impatto della flessibilità gestionale sul valore degli investimenti. I metodi di ricerca includono: analisi logica e comparativa, l'approccio sistemico, modelli economici e matematici. I principali risultati della tesi comprendono la determinazione delle condizioni di applicabilità dei modelli di opzioni reali e l'individuazione di nuove direzioni nell'applicazione del metodo delle opzioni reali relative alla pianificazione degli investimenti e alla gestione del rischio. I risultati della ricerca possono essere utilizzati per analizzare investimenti che sono caratterizzati da un alto livello di rischio e consentono di adeguare la strategia di investimento alle condizioni di mercato.

Abstract

The dissertation explores the use of the real options method in solving various problems. The aim of the work is to develop a tool kit for evaluating real options, which allows for the analysis, planning, and management of investments realised in conditions of high uncertainty. The object of the research is the investment projects of enterprises, implemented in conditions of uncertainty and for different market conditions. The subject of the research is the value of alternative options for action and the impact of managerial flexibility on the value of investments. Research methods include logical and comparative analysis, systems approach, economic and mathematical modelling. The main results of the dissertation include determining the conditions of applicability of models of real options and identifying new directions in the application of the method of real options related to investment planning and risk management. The research results can be used to analyse investments characterised by a high level of risk and allow adjustments to the original strategy in different market conditions.

Contents

Introduction

Criticism of traditional methods of evaluating investment projects began in the 1950s. Projects with flexibility in the 1980s began to be priced similarly to financial options. The term "real option" was coined in 1977 by Myers, who was the first to propose the pricing of real investments by analogy with financial options. A real option is a right, but not an obligation, to make specific investment decisions at specific points in time. Dixit and Pindyck (1994) noticed that most investment decisions have three important characteristics in terms of real options:

- First, the investment is partially or completely irreversible.
- Second, there is uncertainty over the future rewards from the investment.
- Third, investors have some leeway about the timing of investment.

There are a large number of real options, and different projects may have them. The present dissertation illustrates the application of the real options approach in three different areas.

The first chapter discusses applications in the energy sector. In a real options framework, we analyse the behaviour of a large energy producer who can invest in a portfolio of Renewable Energy Source (RES) and *dirty* energy source. Competitive fuel prices challenge the investments in RES. Given a budget constraint, the agent allocates the optimal capacities of both energy instalments and selects the optimal investment time. We use the model to compare the effectiveness of classical support schemes such as Feed-in Tariffs or Green Certificate with respect to forms of taxation of dirty technology such as Carbon Taxes or Carbon

Permits. This chapter proposes a conceptual framework and qualitative analysis to understand which support system enhances the attractiveness of renewable energy investments. The novelty of the chapter lies in the introduction of the dirty option, a fact that the previous literature has ignored so far.

The second chapter investigates the joint effect of uncertainty, competition, and risk-aversion on the optimal time and size of firms in a duopoly. As risk-aversion increases, the leader's alternatives between deterring and accommodating the follower's entry become equivalent. When the leader's role is assigned exogenously, risk-aversion reduces both equilibrium investment sizes and timing. In equilibrium, the leader is always the largest firm in the market. When the leader's role is determined in equilibrium, risk-aversion delays the rent equalization point. At high levels of risk-aversion, both firms invest in the same capacity.

In the third chapter, we investigate the literature on the swing option. Researchers use different approaches to analysis swing options that allow flexibility in planning the supply of oil, gas, electricity, and petroleum products under conditions of uncertainty. The variety of models, as well as the lack of aggregated analysis in this area, suggest the need for a critical review of swing option methodologies and structures. This chapter describes the directions, trends, and designs that we can found in the current academic literature on this topic. The results provide a comprehensive picture of the relevant research, thereby providing researchers with a solid foundation for additional research and guidance for future development.

Each chapter itself is an independent working paper addressing one specific issue encompassing real options and the introduction with the conclusion of the dissertation, which combines the chapters into the holistic dissertation.

Chapter 1

Renewable energy investments, support schemes and the dirty option

1.1 Introduction

The decarbonization of the energy sector, by means of stimuli of investments in Renewable Energy Sources (RES), is a central issue in the agenda of governments worldwide.

According to the International Energy Agency (IEA), renewable electrical capacity increases 50 % (1 220 GW) by 2024, from 2 502 GW in 2018 (IEA, 2019). Nearly two-thirds (64%) of net installations in 2018 were from renewable sources of energy, according to the latest annual Renewables Global Status Report (REN21, 2019). Nevertheless, one cannot deny the fact that fossil fuels made up 82 % of global primary energy in 2015 (Newell et al., 2019). Besides, the Global Energy Outlook (Newell et al., 2019) forecasts that the global energy demand will continue to rise, and most of the demand will be satisfied by fossil-based fuels. In this regard, carbon dioxide emissions from the global energy system are on a path to far exceed international targets of the Paris Agreement.

Due to the fact that the profitability of RES cannot compete with that of traditional fossil-based energy generators, policymakers have been implementing a variety of mechanisms to boost investments in RES. Broadly speaking, such policy mechanisms can be classified into two classes. The first class, which we refer to as subsidies, aims at reducing dioxide emission directly by giving monetary incentives to the energy produced with RES. The second class, which we refer to as carbon pricing, tries to boost investments in RES indirectly by fixing a price on emissions of fossil fuels.

Among the bundle of subsidies implemented worldwide, two prominent examples are Feed-in Tariffs (FiT) and the combination of the quota system and Green Certificates (GC). A FiT is a price-based policy mechanism with which a policymaker offers a fixed price to energy producers per unit of green power sold in the market for a given period of time. Launched for the first time in 1978 with the US National Energy Acts, these support schemes are still prevalent in many countries, and they are widely analysed in the academic literature. The combination of the quota system and green certificates is a quantity-based support scheme also active in many countries, especially Europe. A GC is a tradable asset, whose value fluctuates according to supply and demand, attesting that one unit of power (conventionally 1 MWh) has been generated by RES. Within the scheme, energy producers sell GCs to energy suppliers, who are required to buy a given number of GCs according to the quota system.

According to a recent article in the New York Times (Plumer and Popovich, 2019), as for February 2019, more than 40 countries have set some price on carbon. Important examples of implementations of carbon pricing around the world are Carbon Taxes (CT) and Carbon Emission Trading system, which we refer to simply as Carbon Permits (CP). A CT is a price-based tax for unit of emission of fossils' fuels. A CP is an asset that gives the right to emit one ton of dioxide. CPs are issued by the regulator, who also sets the maximum tons of emissions possible. Once issued, a CP is traded privately, and their value depends on current market conditions.

A recent ongoing debate is which of the two classes of decarbonization mechanisms is preferable in terms of effectiveness, implementation costs, and social fairness. While subsidies are still active in a large part of the world, economists are starting to ask whether or not a shift towards carbon pricing might provide a better decarbonization strategy. Two prominent examples are Bassi et al. (2017) and the *Economist' Statement on Carbon Dividends*, signed by dozens of economists (including 27 Nobel Laureate Economists, 4 former chairs of the Federal Reserve, 15 Former Chairs of the Council of Economic Advisers, and 2 Former Secretaries of the US Department of Treasury) and appeared on the Wall Street Journal in February 2019, in which carbon pricing is described as "the most cost-effective lever to reduce carbon emissions at the scale and speed that is necessary."

In this chapter, we use the Real Options approach of Dixit and Pindyck (1994) to provide a comparative analysis of the effectiveness of carbon pricing and RES subsidies. In addition to the previous book, it is worth mentioning two more works, Trigeorgis (1993) and Trigeorgis (1996), that have firmly entrenched the literature on real options. To do so, we put ourselves on the side of a price-taker energy producer who, given a budget, has to decide the optimal time to invest and the optimal allocation of her budget on power generators based on two different technologies: the green and the dirty technology. We first analyse the baseline case in which no decarbonization schemes are active. Then we examine how the different carbon pricing schemes and RES subsidies affect the optimal allocation and the investment timing of the energy producers. More precisely, we provide two distinct comparisons: the case in which the effectiveness of Feed-in Tariffs is compared with that of carbon taxes; and the case in which green certificates are compared with carbon permits. The choice of these two different comparisons comes from the nature of uncertainty involved in different decarbonization schemes. In fact, while FiT and CT are mainly subject to policy uncertainty (PU), that is, uncertainty deriving from a possible sudden shift in the government's policy, GC and CP are both tradable assets subject to market risk. We focus on two main aspects that are described as follows. From a business perspective, we aim to provide guidance on the evolution of the profitability of RESs. On the other hand, from a regulation point of view, we aim to support regulators in drafting incentive plans in renewable energies.

This chapter belongs to the literature that studies the analysis of investors' behaviour in response to the introduction of decarbonization schemes using the Real Options methodology, recently reviewed in Kozlova (2017) and Trigeorgis and Tsekrekos (2018). In this context, Boomsma et al. (2012) build a model with multiple sources of uncertainty to analyse optimal capacity and investment timing under Feed-in Tariffs and green certificates. They find that while under green certificates, firms invest in larger projects, Feed-in Tariffs promote earlier investments. Also using a setup with various sources of uncertainty, Boomsma and Linnerud (2015) and Adkins and Paxson (2016) analyse investment timing under different subsidies. They both focus on policy uncertainty and use quasi-analytical methods to solve their model. Policy uncertainty is also studied in Dalby et al. (2018), where the authors study a model of Bayesian learning for policy uncertainty. The focus of this prominent stream of the literature is on governments' incentive to RES. Kitzing et al. (2017) valued investments in offshore wind energy in the Baltic Sea amid uncertainties regarding FiT, Feed-In Premiums and tradable GC, and Zhang et al. (2017) focus on the optimal design of subsidies. However, we observe that comparisons between subsidies and carbon pricing are absent and that the mentioned studies ignore the possibility of investing in traditional energy.

Some papers incorporate fuel price uncertainty into the valuation design by analysing various aspects. Siddiqui and Fleten (2010) use a Real Options model to show how a policymaker should allocate funds to boost the development of new technologies. Martinez Cesena and Mutale (2011) analyse a Real Options model for the design of an off-grid photovoltaic generator. In a model with random evolution of fuel prices, Fuss and Szolgayova (2010) analyse how technological uncertainty affects the optimal time of replacement of traditional technologies with new less fuel-intensive power generators. Along the same line, but without technological uncertainty, Li et al. (2015), Xian et al. (2015) build a real options model to analyse the optimal investment time in new fuel-based technologies. This piece of literature recognizes fuel price as an essential factor driving investors' choices, although the focus on decarbonization schemes is absent.

With this chapter, we aim to complement the literature on Real Options for energy investments in two aspects. First, we give the decision-maker the option of investing not only in renewable energy but also in conventional energy, thus giving more flexibility to the decision-makers. We refer to this additional flexibility as *the Dirty Option*. To the best of our knowledge, this is done for the first time in the described context, bringing the analysis closer to real investment problems. Last, we provide a comparative analysis of the effectiveness of subsidies and carbon pricing. Consequently, we aim to determine at what point the RES become so attractive (or profitable) as conventional energy. Besides, we intend to present a practical model for the evaluation of the project that is convenient and understandable for both researchers and practitioners.

The chapter is organized as follows. In section 2, we describe our model. We explore the investment timing and capacity selection options in Section 3. In Section 4, we discuss the results of sensitivity analysis. Finally, in section 5, we conclude.

1.2 Model's setup

We use a continuous-time infinite-horizon real options framework, in which a price-taking energy producer contemplates the installation of new energy production plants. Two alternative technologies are available: the traditional (dirty) fossil-based technology, denoted by D and the renewable (green) technology such as wind or solar power, denoted by G. Given an available budget B, the firm's problem consists in determining the optimal capacity in dirty (D) and green (G) energy source, denoted by x_D and x_G, respectively, and the optimal time to expand the production capacity. We measure capacity in terms of power, that is one unit of capacity corresponds to one Megawatt (MW). Let I_h be the cost of installation of one unit of power of technology h, $h = D, G$. The equation $I_D x_D + I_G x_G = B$ describes the budget constraint faced by the producer. Without loss of generality, we assume for simplicity that once installed, both technologies are capable of producing energy for T years. After T years of use, the power plants do not produce efficiently and are dismissed.

The production function of dirty technology is $Q_D(x_D) = A_D \cdot h_D \cdot x_D$, where A_D is total hours in a year, h_D is the capacity factor[1] for traditional technology. The production function of green technology is $Q_G(x_G) = A_G \cdot h_G \cdot x_G^{\gamma}$, where A_G is total hours in a year, h_G is the capacity factor for renewable technology and $\gamma \in (0, 1)$. Our assumption about the concavity of the production function of green technology is in line with the literature. For instance, Boomsma et al. (2012) justify diminishing marginal production resulting from increasing capacities by wake effects. We refer to Boomsma et al. (2012) for further details.

Following the literature on Real Options, we assume that electricity prices (E_t) and fuel prices (F_t) follow two GBM [2]:

[1]The net capacity factor is the unitless ratio of actual electrical energy output over a given period of time to the maximum possible electrical energy output over that period. The capacity factor is defined for any electricity-producing installation, such as a fuel-consuming power plant or one using renewable energy, such as wind or the sun.

[2]For the sake of brevity, we later suppress the subscript t for both the price of electricity (E_t) and the price of fuel (F_t) whenever suitable.

$$\frac{dE_t}{E_t} = \mu_E dt + \sigma_E dW^E(t); \\ \frac{dF_t}{F_t} = \mu_F dt + \sigma_F dW^F(t), \tag{1.1}$$

where μ_E, μ_F and σ_E, σ_F are the corresponding instantaneous rates of return and volatilities, respectively. $W^E(t)$ and $W^F(t)$ are two standard correlated Brownian motions, with correlation coefficient ρ^{EF}.

1.2.1 Decarbonization schemes

We denote by G_t the instantaneous value of a generic subsidy and use two different stochastic representations to distinguish between FiTs and GCs. The stochastic process that models a FiT is indeed a piecewise constant process, reflecting the fact that a FiT is subject to changes only in response to a change in the support policy. We follow the relevant real options literature and specify the model for a FiT under policy risk by means of a continuous-time Markov chain with two states, $\{G_{Good}, G_{Bad}\}$, of which G_{Bad} is absorbing and transition rate λ_G. For additional details about the computation of the relevant quantities, we refer to the Appendix A.2. Conversely, given that GCs are freely traded, their value changes continuously over time according to the prevalent market conditions. Thus, we use a GBM to model the firm's income due to green certificates, that is

$$dG_t = \mu_G G_t dt + \sigma_G G_t dW_t^G,$$

with μ_G, σ_G being the instantaneous rate of return and the volatility, respectively. The Brownian motion governing the dynamics is correlated to W^E, W^F, with correlation coefficients ρ^{GE}, ρ^{GF}, respectively.

In the same way, we denote by C_t the cost of a generic carbon price and use two different probabilistic models to differentiate between CTs and CPs. Carbon taxes are subject to policy risk only: they change values only due to a policy change. We use a continuous-time Markov chain again with state-space $\{C_{Good}, C_{Bad}\}$, starting in C_{Good}, with C_{Bad} as an absorbing state and transition rate λ_C from C_{Good} to C_{Bad}. Finally, the value of carbon permits fluctuates according to market conditions. Thus, we model CPs by means of a different GBM:

$$dC_t = \mu_C C_t dt + \sigma_C C_t dW_t^C,$$

with the usual interpretation of parameters μ_C, σ_C and driving Brownian motion W^C correlated to W^E, W^F, with correlation coefficients ρ^{CE}, ρ^{CF}, respectively.

1.2.2 The decision problems

Let us define the instantaneous profits as usual, by:

$$\pi(E, F, G, C; x_G, x_D) = Q_G(x_G)(E + G) + Q_D(x_D)(E - F - C). \quad (1.2)$$

The decision problem of the investor consists of choosing the optimal capacities, x_G, x_D, and the optimal time to invest, τ, so as to maximize the net present value of the future profits. The optimal capacities must lie in the admissible set $\mathcal{I} = \{x_G, x_D \geq 0, I_G x_G + x_D I_D = B\}$, which describes the budget constraint. Different types of decarbonization schemes imply a different decision problem to be solved. In particular, we have several cases.

The first case, which we refer to as the Baseline case, is the one in which no decarbonization scheme is active in the country. In this case, we set $G_t = 0$ and $C_t = 0$ and the decision problem is:[3]

$$\begin{aligned} &V^B(E, F) = \\ &\max_{\tau \geq 0} \max_{x_G, x_D \in \mathcal{I}} \mathbb{E}_{E,F} \left[\int_\tau^{\tau+T} e^{-rs} \pi(E_s, F_s, 0, 0; x_G, x_D) ds \right] - B = \qquad (1.3) \\ &\max_{\tau \geq 0} \max_{x_G, x_D \in \mathcal{I}} \mathbb{E}_{E,F} \left[e^{-r\tau} \right] \left(L^B(E, F; x_G, x_D) - B \right), \end{aligned}$$

[3]We use $\mathbb{E}_y(\cdot)$ to denote the conditional expectation of a stochastic process starting at y.

where the net present value of future profits is given by:

$$\begin{aligned}&L^B(E,F;x_G,x_D) = \left(Q_G(x_G)+Q_D(x_D)\right)E\bar{r}_T(\mu_E,0)-\\&Q_D(x_D)F\bar{r}_T(\mu_F,0),\end{aligned} \tag{1.4}$$

and $\bar{r}_T(\mu,\lambda)=\frac{1-e^{-(r-\mu+\lambda)T}}{r-\mu+\lambda}$. Next, we consider the case in which a Feed-in tariff is active. In this case, we impose G_t to follow the continuous-time Markov chain described in the previous subsection and set $C_t=0$:

$$\begin{aligned}&V^{FiT}(E,F) =\\&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,G_{Good}}\left[\int_\tau^{\tau+T}e^{-rs}\pi(E_s,F_s,G_s,0;x_G,x_D)ds\right]-B=\\&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,G_{Good}}\left[e^{-r\tau}\right]\left(L^{FiT}(E,F;x_G,x_D)-B\right),\end{aligned} \tag{1.5}$$

where

$$\begin{aligned}&L^{FiT}(E,F;x_G,x_D)=\\&\quad\left(Q_G(x_G)+Q_D(x_D)\right)E\bar{r}_T(\mu_E,0)-Q_D(x_D)F\bar{r}_T(\mu_F,0)+\\&\quad Q_G(x_G)\left(G_{Good}\bar{r}(0,\lambda_G)+G_{Bad}\left(\bar{r}(0,0)-\bar{r}(0,\lambda_G)\right)\right).\end{aligned} \tag{1.6}$$

Next, we consider Green certificates, by setting $dG_t=\mu^G G_t dt+\sigma^G G_t dW^G(t)$ and $C_t=0$:

$$\begin{aligned}&V^{GC}(E,F,G) =\\&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,G}\left[\int_\tau^{\tau+T}e^{-rs}\pi(E_s,F_s,G_s,0;x_G,x_D)ds\right]-B=\\&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,G}\left[e^{-r\tau}\right]\left(L^{CG}(E,F,G;x_G,x_D)-B\right),\end{aligned} \tag{1.7}$$

where

$$\begin{aligned}&L^{GC}(E,F,G;x_G,x_D)=\\&\quad\left(Q_G(x_G)+Q_D(x_D)\right)E\bar{r}_T(\mu_E,0)-Q_D(x_D)F\bar{r}_T(\mu_F,0)+\\&\quad Q_G(x_G)G\bar{r}_T(\mu_G,0).\end{aligned} \tag{1.8}$$

Next, we consider the case in which a Carbon Tax. In this case, we impose C_t to follow the continuous-time Markov chain described in the previous subsection and set $G_t = 0$:

$$
\begin{aligned}
&V^{CT}(E,F) = \\
&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,C_{Good}}\left[\int_{\tau}^{\tau+T} e^{-rs}\pi(E_s,F_s,0,C_s;x_G,x_D)ds\right] - B = \\
&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,C_{Good}}\left[e^{-r\tau}\right]\left(L^{CT}(E,F;x_G,x_D) - B\right),
\end{aligned}
\tag{1.9}
$$

where

$$
\begin{aligned}
L^{CT}(E,F;x_G,x_D) &= \\
&(Q_G(x_G)+Q_D(x_D))\,E\bar{r}_T(\mu_E,0) - Q_D(x_D)F\bar{r}_T(\mu_F,0) - \\
&Q_D(x_D)\left(C_{Good}\bar{r}(0,\lambda_G) + C_{Bad}\left(\bar{r}(0,0)-\bar{r}(0,\lambda_G)\right)\right).
\end{aligned}
\tag{1.10}
$$

Finally, we consider Carbon Permits for the last case, thus setting $dC_t = \mu^C C_t dt + \sigma^C C_t dW^C(t)$ and $G_t = 0$:

$$
\begin{aligned}
&V^{CP}(E,F,C) = \\
&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,C}\left[\int_{\tau}^{\tau+T} e^{-rs}ds\pi(E_s,F_s,0,C_s;x_G,x_D)\right] = -B \\
&\max_{\tau\geq 0}\max_{x_G,x_D\in\mathcal{I}}\mathbb{E}_{E,F,C}\left[e^{-r\tau}\right]\left(L^{CP}(E,F,C;x_G,x_D) - B\right),
\end{aligned}
\tag{1.11}
$$

where

$$
\begin{aligned}
L^{GC}(E,F,C;x_G,x_D) &= \\
&(Q_G(x_G)+Q_D(x_D))\,E\bar{r}_T(\mu_E,0) - Q_D(x_D)F\bar{r}_T(\mu_F,0) - \\
&Q_D(x_D)G\bar{r}_T(\mu_C,0).
\end{aligned}
\tag{1.12}
$$

1.3 Numerical study

In this section, we perform a series of numerical comparisons between the two philosophically different ways of boosting investments in green energy sources: RES subsidies and carbon pricing. To do so, we focus on an energy producer's the point of view, who observes the market conditions and makes investment decisions.

We consider an investment in two types of energy where the project life is 20 years. The investment costs of wind power plants installation are set equal to $I_G = 1600$ T euros/MW years and $I_C = 900$ T euros/MW. These numbers are the median cost for Europe. The risk-adjusted real discount rate is set to $r = 5.0$ %, reflecting an inflation rate of $2,5\%$. We use μ_E=μ_F=0 in the price processes, which implies that these prices likewise grow with the general price level. The electricity price volatility and the fuel price volatility equal 0.06; the corresponding correlation coefficient is estimated to 0.7. The values of the remaining parameters are presented in Appendix A.1.

We first study the base case, that is a situation in which investors have not incentives. We then examine how investors change their decision if some support scheme is present. As far as the numerical techniques are concerned, we use Monte Carlo simulation in conjunction with Least-Square regression to obtain investment values. Besides, for all two-dimensional problems we use a finite difference scheme to visualize investment regions.

1.3.1 Baseline

Our base case assumes no regulation, such as subsidy payments or carbon pricing. Figure 1.1 shows the investment decision of an energy producer for all possible combinations of electricity and fuel prices. We restrict attention to the cases in which energy prices are higher than fuel costs. This means that the part of the graphs where $E < F$ is not taken into consideration.[4] Figure 1.1 is divided into three sub-regions. The white

[4]In our setup, this assumption seems to be reasonable. However, we acknowledge that in a real energy market, energy price can be lower than fuel costs. This is due to the so called *marginal pricing*, for which the energy price is the maximum cost

area represents the no-investment region. The colored area of the graphs indicates combinations of energy prices and fuel costs in which the producer invests. The color of the sector responsible for investment answers the question of how much green energy needs to be invested. Green color indicates investments in green energy only, while the saturation of the shade of blue speaks of the predominance of investments in dirty energy.

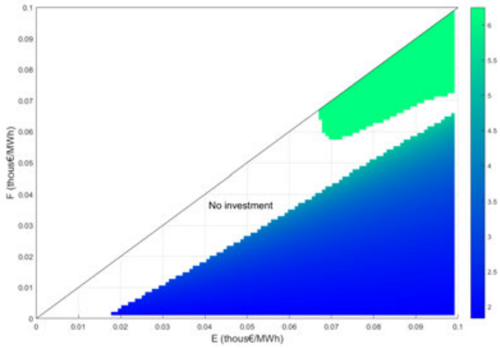


Figure 1.1: The base case.

We emphasize that we get a graph relative to electricity and fuel prices, each point of which corresponds to the optimal value of x_G. To better explain Figure 1.1, we take two extreme points into consideration. We will consider the investor's decisions when electricity prices equal to 40 and 80 euros. At a low price of electricity, if the fuel cost is sufficiently high, the investor does not invest. Upon reaching a sufficiently low fuel price, the producer invests most of the budget in dirty energy. When electricity price is high (in our case, 80 euros), the investment decision-making system changes. If the fuel cost is high enough, the entire budget is invested exclusively in RES.

We observe a region where the investor finds optimal not to invest at all. The colorless part of the graph represents that. Given the level of energy price, the fuel cost is not low enough; in this area, the producer does not make any investment but wait for more favorable market

for producing a given amount of power. Handling such situations requires a setup in which energy price is endogenous as in Aïd et al. (2013) and goes far beyond the scope of the present chapter.

Table 1.1: Investment values of base case for different levels of prices, T euros

F \ E	0.05	0.06	0.07
0.01	21110.60	29331.12	37559.98
0.02	11404.14	19485.98	27632.68
0.03	3115.89	10223.31	18104.92
0.04	815.05	3129.99	9358.30
0.05	449.30	1339.24	3485.56

conditions. Such regions are typical of a real option framework. It is also worth to note that there is a gap between the zones of investment in both types of energy and only green. In such area, energy prices are very high. However, fuel cost is neither sufficiently low to justify a huge investment in dirty energy, nor high enough to boost investments in green energy sources.

In Table 1.1, we present the value of the investments in some representative cases. The table shows that the investment values decrease with increasing fuel prices and increase with increasing electricity prices.

1.3.2 Feed-in Tariff versus Carbon Taxes

Capacity choice. We define

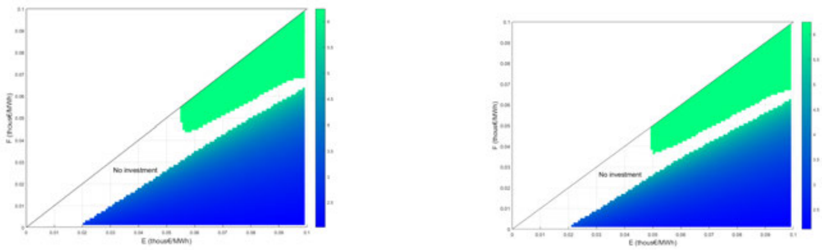
$$\begin{aligned} W(x_G, x_D) = & E\tilde{r}_E^T(Q_D(x_D) + Q_G(x_G)) - F\tilde{r}_F^T Q_D(x_D) + \\ & G\tilde{r}Q_G(x_G) - C\tilde{r}Q_D(x_D) - B. \end{aligned}$$

The problem of maximizing $W(x_G, x_D)$ under the budget and the non-negativity constraint posses a unique solution given by $(x_G^*, \frac{B-I_G x_G}{I_D})$, where

$$x_G^* = \begin{cases} \min\left(\left(\frac{A_G \gamma I_D (E\tilde{r}_E^T + G\tilde{r})}{A_D I_G (E\tilde{r}_E^T - F\tilde{r}_F^T - C\tilde{r})}\right)^{\frac{1}{1-\gamma}}, \frac{B}{I_G}\right) & E > \frac{C\tilde{r} + F\tilde{r}_F^T}{\tilde{r}_E^T} \\ \frac{B}{I_G} & E \leq \frac{C\tilde{r} + Fr_F^T}{r_E^T} \end{cases} \tag{1.13}$$

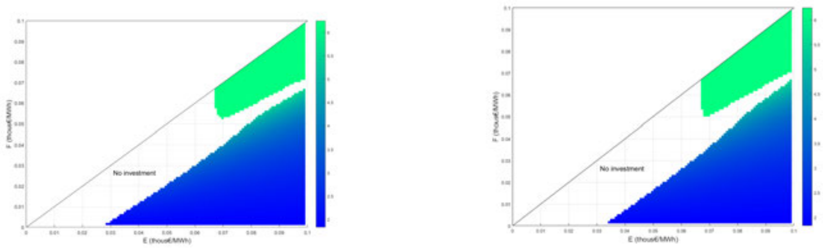
We investigate how each support scheme or carbon pricing affects the behaviour of an energy producer. The similarity between FiT and

CT is noted because both are price-based instruments. We take Feed-in Tariff and carbon tax equal to either 10 or 15 euros/MW Hour.



(a) (b)

Figure 1.2: Feed-in tariff equal to: (a) 10 euros/MW Hour; (b) 15 euros/MW Hour.



(a) (b)

Figure 1.3: Carbon Tax equal to: (a) 10 euros/MW Hour; (b) 15 euros/MW Hour.

We first analyse the effects of FiT and CT on the investment region, via a visual comparison with the baseline investment region in Figure 1.1. Figures 1.2(a) and 1.2(b) depict the investment region when a FiT of 10 and 15 euro, respectively, is active. When compared with the baseline case, in both panels, we observe an enlargement of the green area. In such cases, the firm invests in green energy only at lower energy prices, provided that the fuel cost is low enough. On the other hand, the remaining part of the colored area of the graph remains basically unchanged. Figures 1.3(a) and 1.3(b) depict the investment region when a CT of 10 and 15 euro, respectively, is active. In this case, we observe a restriction of the area in which the firm invests predominantly in dirty energy. Carbon taxes discourages investments in dirty technology but does not boost investments in green energy.

Table 1.2: Investment values under Feed-in Tariff, percentage change of investment value

Feed-in Tariff	10 euros/MW Hour			15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-7.69	-5.79	-4.65	-11.11	-8.43	-6.82
0.02	-10.36	-7.09	-5.44	-14.50	-10.17	-7.89
0.03	-10.23	-8.47	-6.30	-11.27	-11.56	-8.97
0.04	34.41	1.75	-5.93	113.62	15.94	-7.66
0.05	124.56	97.45	27.75	287.53	164.89	53.60

Table 1.3: Investment values under Carbon Tax, percentage change of investment value

Carbon Tax	10 euros/MW Hour			15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-32.55	-23.96	-18.99	-48.82	-35.94	-28.48
0.02	-48.83	-31.25	-23.17	-70.41	-46.88	-34.76
0.03	-54.71	-44.27	-29.32	-69.85	-63.78	-43.99
0.04	-33.94	-40.04	-37.23	-44.22	-53.83	-54.53
0.05	-9.72	-12.66	-21.14	-12.35	-15.55	-24.14

Feed-in-Tariffs and Carbon Taxes impact differently also in terms of the value of the investment. In Table 1.2, we report the percentage changes - with respect to the base case - of the investment values when a FiT is present in the market. We note that for the FiT to have a positive impact on the value of the investment it is necessary to have a sufficiently high marginal profit, that is the difference between energy prices and fuel cost. However, the percentage change of the investment values decreases as the marginal profit increase. In Table 1.3, we report the percentage change of the investment value in the presence of a Carbon Tax. The introduction of a CT reduces the value of the investment in all the cases considered.

1.3.3 Green certificates versus Carbon permits

Capacity choice. We define

$$\begin{aligned} W(x_G, x_D) =& E\tilde{r}_E^T(Q_D(x_D) + Q_G(x_G)) - F\tilde{r}_F^T Q_D(x_D) + \\ & G\tilde{r}_G^T Q_G(x_G) - C\tilde{r}_C^T Q_D(x_D) - B. \end{aligned}$$

The problem of maximizing $W(x_G, x_D)$ under the budget and the non-negativity constraint posses a unique solution given by $(x_G^*, \frac{B - I_G x_G}{I_D})$, where

$$x_G^* = \begin{cases} \min\left(\left(\frac{A_G \gamma I_D (E\tilde{r}_E^T + G\tilde{r}_G^T)}{A_D I_G (E\tilde{r}_E^T - F\tilde{r}_F^T - C\tilde{r}_C^T)}\right)^{\frac{1}{1-\gamma}}, \frac{B}{I_G}\right) & E > \frac{C\tilde{r}_C^T + F\tilde{r}_F^T}{\tilde{r}_E^T} \\ \frac{B}{I_G} & E \leq \frac{C\tilde{r}_C^T + F r_F^T}{r_E^T} \end{cases} \tag{1.14}$$

Here we compare the investment value in the presence of green certificates and carbon permits. For GC, we use starting values of the certificate equal to either $G_0 = 10$ or $G_0 = 15$. For CP, we use C_0 either equal to 10 or 15. In Tables 1.4 and 1.5, we report percentage changes of investment values with respect to the baseline case.

Table 1.4: Investment values under Green certificates, percentage change of investment value

Green certificates	G_0 =10 euros/MW Hour			G_0 = 15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-7.84	-6.01	-4.82	-11.25	-8.65	-6.98
0.02	-10.51	-7.28	-5.60	-14.64	-10.35	-8.05
0.03	18.45	-8.42	-6.44	14.81	-11.34	-9.10
0.04	112.65	31.12	-5.56	141.90	35.80	-7.22
0.05	181.89	99.39	42.67	310.52	164.51	57.66

Table 1.5: Investment values under Carbon permits, percentage change of investment value

Carbon permits	C_0 = 10 euros/MW Hour			C_0 = 15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-46.03	-33.71	-26.56	-67.24	-49.73	-39.40
0.02	-65.35	-47.56	-34.60	-78.07	-67.35	-50.83
0.03	-52.93	-60.35	-48.30	-66.24	-73.33	-65.32
0.04	-13.64	-41.34	-55.00	-28.33	-52.04	-65.88
0.05	21.32	1.22	-21.61	29.06	-0.43	-23.75

The main difference between green certificates and carbon permits is what they offset. Where CP help reduce greenhouse gas emissions, GC offset electricity use from renewable sources. Carbon permits provide certainty of abatement quantity but render the price per unit of abatement uncertain. Green certificates significantly increase investment values at low electricity prices and high fuel prices, while in the opposite situation (high electricity prices and low fuel prices), the investment values are lower than in the baseline. Carbon permits, on the other hand, reduce the investment value of the firm. This seems in line with the results of the previous subsection about the Carbon Tax.

1.4 Sensitivity analysis

In this section, we verify the robustness of the previous results with respect to change in some crucial parameters. More specifically, we are interested in two main factors: policy risk and the budget available to the firm. We proceed as usual, by looking at the differences between Feed-in Tariffs versus Carbon Taxes and Green Certificates versus Carbon Permits.

1.4.1 Feed-in tariff versus carbon taxes

Introducing policy risk In section 1.3, we examined the case where neither the support scheme nor the carbon pricing could be altered during the life of the facilities installed. Here, we introduce policy uncertainty by allowing for a random revision of the support schemes. This seems to be reasonable, given the high speed of technological development in renewable energy sources. We assume that such revisions arrive at intensity $\lambda = 0.1$, implying on average a shift in the scheme every 10 years. We take the initial value of the FiT equal to 15 euros/MW Hour (the good state) and assume that this value can be lowered to 10 euros/MW Hour (the bad state) at some random point in time. For Carbon Taxes, we take the initial value to 10 euros/MW Hour (the good state). Such value is subject to a change to 15 euros/MW Hour at a random point in time.

Table 1.6 presents the investment values obtained in this new setup. The results are qualitatively similar to the case without policy uncertainty, providing robustness of our analysis with respect to policy uncertainty. However, we see that policy uncertainty prejudices investors' readiness to invest in RES and consequently reduces the effect of decarbonization schemes. At the same time, the penalizing effects of a Carbon Tax in terms of appeal of the investment are slightly reduced.

Increasing the budget Here, we are interested in considering an increase in investment up to 100 million to analyse changes in the optimal patterns. Tables 1.8 and 1.9 present percentage changes of investment values in such cases, with respect to the base case, whose investment

Table 1.6: Investment values under FiT and Carbon Tax with policy uncertainty ($\lambda = 0.1$), percentage change of investment value

Support schemes	FiT			Carbon Tax		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-9.44	-7.14	-5.75	-40.67	-29.94	-23.73
0.02	-12.52	-8.67	-6.69	-60.78	-39.05	-28.95
0.03	-11.23	-10.11	-7.68	-63.11	-54.89	-36.64
0.04	64.47	6.99	-6.90	-39.67	-47.56	-46.34
0.05	193.23	131.24	40.67	-11.22	-14.38	-23.84

values are in Table 1.7. We consider no policy uncertainty ($\lambda = 0$).

Table 1.7: Investment values of base case for different levels of prices, $B = 100$ million

	Investment values, T euros		
F \ E	0.05	0.06	0.07
0.01	319091.03	424306.66	529530.62
0.02	212389.46	317466.41	422608.22
0.03	106685.80	211208.63	316085.35
0.04	27204.38	106364.16	210343.62
0.05	7904.62	32639.31	106487.02

Once again, the results are qualitatively consistent with those presented in Section 1.3. However, the effects of FiT -both positive and negative- are dramatically reduced. This highlights the fact that the smaller the firm, the less pronounced the effect of Feed-in-Tariffs, at least in terms of investment values. On the other hand, the loss of investment value in the presence o a Carbon Tax is much more pronounced.

1.4.2 Green certificates versus carbon permits

Increasing the budget We set the budget available equal to 100 million. The effects of a higher budget when Green Certificates are active on the markets is highlighted in Table 1.10. As in the case of Section

Table 1.8: Investment values under Feed-in Tariff, percentage change of investment value, $B = 100$ million

Feed-in Tariff	10 euros/MW Hour			15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-0.51	-0.40	-0.33	-0.73	-0.58	-0.48
0.02	-0.56	-0.44	-0.36	-0.78	-0.62	-0.52
0.03	-0.30	-0.41	-0.36	-0.30	-0.56	-0.51
0.04	0.80	0.12	-0.26	1.63	0.34	-0.34
0.05	4.54	1.57	0.54	9.17	2.98	0.97

Table 1.9: Investment values under Carbon Tax, percentage change of investment value, $B = 100$ million

Carbon Tax	10 euros/MW Hour			15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-32.55	-24.52	-19.66	-48.83	-36.77	-29.50
0.02	-48.29	-32.47	-24.47	-72.44	-48.71	-36.70
0.03	-89.43	-48.07	-32.37	-121.98	-72.10	-48.55
0.04	-201.68	-88.98	-47.77	-151.72	-116.43	-71.65
0.05	-193.52	-146.53	-85.86	-162.72	-119.05	-108.58

1.3, we note that GCs are particularly able to stimulate investment in RES. In fact, the results are qualitatively similar. In addition, Table 1.10 shows that a higher budget increases the incentive of the firm to invest in RES, as the loss in investment value in the cases observed in Section 1.3 is much less pronounced than the case of a lower budget. On the other hand, the negative effects of Carbon Permits already observed in the previous section are more pronounced when the firm is willing to invest more, as shown in Table 1.11.

Table 1.10: Investment values under Green certificates, percentage change of investment value, $B = 100$ million

Green certificates	G_0 =10 euros/MW Hour			G_0 =15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-0.69	-0.57	-0.51	-0.96	-0.75	-0.70
0.02	-0.70	-0.67	-0.53	-0.93	-0.77	-0.68
0.03	-0.19	-0.60	-0.55	0.00	-0.75	-0.68
0.04	69.16	1.83	-0.47	66.76	1.10	-0.50
0.05	173.28	66.60	7.95	157.18	63.69	7.22

Table 1.11: Investment values under Carbon permits, percentage change of investment value

Carbon permits	C_0 =10 euros/MW Hour			C_0 =15 euros/MW Hour		
F \ E	0.05	0.06	0.07	0.05	0.06	0.07
0.01	-33.55	-25.29	-20.36	-50.12	-37.82	-30.41
0.02	-49.81	-33.52	-25.29	-69.21	-50.15	-37.85
0.03	-61.18	-49.00	-33.55	-73.98	-67.05	-49.96
0.04	-35.76	-54.31	-47.81	-58.00	-68.52	-62.38
0.05	-7.43	-30.37	-48.60	-35.89	-51.38	-63.01

1.5 Conclusion

This chapter presents a real options framework to value investment timing and capacity choice of investments in energy facilities. Our main focus is on contributing to the debate on which renewable energy support scheme does the best job in boosting investment in renewable energy sources.

On the modelling side, our study differs from the relevant literature in one major aspect: We give the investor the opportunity to invest also in traditional energy sources. We call this opportunity the Dirty Option.

In our analysis, we observe the effectiveness of FiT in driving green energy investment. At the same time, CT is holding back investment in RES. Difficulties and complexities in the development of GC and CP may explain why these incentives are not so common. However, the incentive effect of a green certificate is comparable to FiT.

Chapter 2

Strategic capacity choice with risk-averse firms

2.1 Introduction

In a world where new investment possibilities arrive at rates never experienced, studying and understanding investment problems assume high practical relevance. For instance, in the telecommunication sector, the fifth-generation mobile technology (usually know as 5G) has been just launched, opening up new investment opportunities and dilemmas for both services and hardware providers. As auctions for national's band spectrum open, telecommunication services providers engage in costly races to achieve a future strategic advantage over competitors. For example, the Italian Government earned almost €6.5 billion against €2.5 billion planned. However, the automobile sector exhibits a different investment pattern. The industry has been experiencing only recently investments in electrical engines and a variety of hybrid vehicles though Toyota launched its first hybrid car over twenty years ago.

Since the seminal works of McDonald and Siegel (1986), Dixit and Pindyck (1994), the new paradigm of investments under uncertainty has inspired an enormous number of theoretical and applied papers on real options. Strategic real options theory, the sub-field of real options theory dealing with investment under uncertainty and competition, originated with Smets (1991), Trigeorgis (1991), is among the most successful and currently most active streams of research. Recent advances add to the classical investment problem not only competition but also a new strategic decision variable, namely the scale of the investment, that extends the range of the predictions of real options theory.

The majority of real options models makes predictions based on the assumption that agents are risk-neutral. Working in a risk-neutral setup gives the double advantage of simplifying the problems involved and obtaining a result that is comparable with the literature. However, in reality, we observe a variety of behavioural patterns that seem to favor the risk-aversion hypothesis. This assertion is based not only on anecdotal evidence. Recent empirical investigations test predictions based on real options theory and study when agents exercise American options, finding that observed investment practice might contrast predicted behaviour. For instance, Linnerud et al. (2014) test the effects of policy uncertainty on investments on small hydropower projects. In contrast to what real option predicts, authors find that non-professionals did not recognize the possibility of waiting to increase the project value. They conclude that, concerning that specific class of investors, *the assumptions made by real options theory with respect to investors' preferences, characteristics, and behaviour are less realistic.* Although not in the context of pure investment behaviour, the recent paper of Carpenter et al. (2015) provides further evidence. In that paper, authors analyse the exercise of Employees Stock Options in a unique database of over 100 firms and find the behaviour of professionals in sharp contrast with risk-neutrality.

The present chapter studies the joint effect of uncertainty, competition, and risk-aversion on the equilibrium timing and capacity choice. Two firms contemplate the possibility of investing in a single-product production plant. Instantaneous Hyperbolic Absolute Risk Aversion (HARA) utility functions model the risk-averse propensity of the two firms. Each firm has to decide both the optimal time to invest and the capacity of the plant. Following the current literature, we distinguish between two cases: First, we fix a priori which firm is the first to invest. In this case, the resulting equilibrium is said to be non-preemptive. Then we analyse the case in which the question concerning which of the two firms is the first to invest is part of the problem itself. In this case, both firms have economic incentives to invest first due to the presence of a first-mover advantage. The resulting equilibrium is said to be preemptive.

The interaction between uncertainty, competition, and risk-aversion produces the following results. First, risk-aversion reduces equilibrium capacities (in both preemptive and non-preemptive equilibria).

Second, as risk-aversion increases, the incentive of the leader to delay follower's investment as much as possible diminishes. Two forces contribute to this effect. First, the follower's investment threshold is a decreasing function of the player's attitude toward risk (see Chronopoulos et al., 2013). Thus, as risk-aversion increases, deterring the follower's entry becomes less appealing, because of the exogenous reduction of the leader's extra profit during the period of temporary monopoly. Third, a risk-averse leader reduces the size of the investment. This reduces further the leader's benefit due to the temporary monopoly. In the end, for a sufficiently high level of risk-aversion, deterring and accommodating the follower's investment become almost indistinguishable. Consequently, the two strategies available produce the same investment size.

Forth, when firms' roles are preassigned, the large investment required by the deterrence strategy increases the probability of investment failure, due to the sunk costs of entry. The extra-utility from profits during the period of monopoly is not sufficient to compensate for the loss of utility the leader suffers from her risk exposure. As the players' risk-aversion increases, the latter effect becomes dominant. The leader becomes less aggressive, thus reducing the over-investment effect found in Huisman and Kort (2015). Moreover, investing a lower amount of money reduces the market size necessary to enter the market. As a result, the higher the risk-aversion, the lower the leader's investment threshold. Further, higher risk-aversion makes the follower willing to invest earlier. In equilibrium, the leader is always the largest firm in the market.

Fifth, when firms' roles are determined in equilibrium, we observe the following patterns. The preemption point is increasing with risk-aversion. Thus, risk-aversion has a positive relationship with investment delay. The question for which firm is the largest in equilibrium has no definitive answer but depends on the parametrization used. However, when risk-aversion is high, both firms choose the same investment size.

Lastly, in one of the asymmetric case we consider, we find a region in which the leader cannot implement the available strategies. In this region, the Leader only alternative is to wait until the value of the state variable allows the implementation of the strategies.

The rest of the chapter is structured as follows. In subsection 2.1.1, we review the literature related to this chapter. Section 2.2 introduces the basic setup and firms' preferences and solves the combined optimal stopping problems with endogenous capacity choice involved in the subsequent analysis. Section 2.3 focuses on the equilibrium solution when the role assigned to each player in the market is given ex-ante, while Section 2.4 analyses the case in which the role played by the firm is part of the equilibrium solution. Finally, in Section 2.5, we conclude.

2.1.1 Related literature

The literature on real options games is so broad that a complete review of the topics studied is outside the objective of this chapter. We limit the discussion to the streams of research that are most relevant for the present chapter and refer to Azevedo and Paxson (2014) for comprehensive literature reviews.

This chapter is related to several research lines. The first line deals with capacity and timing decisions in investments under uncertainty originated from Dangl (1999), Bar-Ilan and Strange (1999). In this context, recent papers analyse different aspects. Chronopoulos et al. (2013) look at the effects of operational flexibility; Hagspiel et al. (2016), De Giovanni and Massabò (2018) focus on the impact of volume flexibility. Koussis et al. (2007) focus on time-to-learn and learning-by-doing, while Chronopoulos and Siddiqui (2015) analyse the replacement of old technologies. The analysis provided in the present chapter does not consider either operational nor volume flexibility. However, it adds to this stream of the literature by introducing the combined effect of competition and risk-aversion in a capacity decision problem.

The second research line analyses the effect of risk-aversion on the classical real options paradigms. In this vein, Hugonnier and Morellec (2007) is the first paper to show that risk-aversion causes a dramatic reduction of the investment value due to a delay in the optimal time to invest. Chronopoulos et al. (2011) suggest that operational flexibility alleviates the effects of risk-aversion. Chronopoulos and Lumbreras (2017) contribute to this stream by analysing the impact risk-aversion on sequential investments in new technologies. The present study adds to this stream of literature by providing novel predictions on the effect of risk-aversion in a strategic setup under endogenous capacity choice.

The present study is also related to the recent and fast-growing stream of the literature that focuses on the strategic investment problem under endogenous capacity selection, originated by the seminal paper of Huisman and Kort (2015). Under this line of research, Lavrutich et al. (2016) analyse the joint effect of uncertainty about both market growth and additional market players. Aïd et al. (2017) consider a game of capacity investment in power generation. Lavrutich (2017) also add exit to the basic model. The contribution that the present chapter offers to this line of research is twofold. On the one hand, investment models with competition, endogenous capacity choice and endogenous product pricing are particularly difficult to analyse due to the intrinsic coupled nature of such problems. In this direction, in the proof of Proposition 3, we show how the implicit function theorem can transform the problem of finding optimal capacities into a nonlinear equation. On the other hand, the present chapter introduces a more general framework where firms are characterized by varying degrees of risk-aversion. Under this setup, most of the effects that the study highlights are, to the best of our knowledge, new.

The two papers most related to the present study are Chronopoulos et al. (2014), Huisman and Kort (2015). Chronopoulos et al. (2014) analyse the impact of risk-aversion on the classical strategic investment problem, taking capacity as given. Huisman and Kort (2015) consider the capacities of both players as a decision variable in a risk-neutral setup. The present study examines both risk-aversion and endogenous capacity decision at the same time, thus providing a bridge between the existing research gap.

2.2 Model and strategies

We consider an industry consisting of two firms. Both firms contemplate undertaking an investment in a production facility. The decision problem of both firms involves the timing of the investment and the capacity of the production plant. Firms start production only after installing capacity. We suppose that investment costs are linear in capacity size, with marginal cost denoted by c, sunk and symmetric to both firms. Firms are supposed to produce always up to capacity.

The price at time t of the product is given by:

$$p(Q, X_t) = X_t(1 - \eta Q), \tag{2.1}$$

where Q is the total capacity of the industry, while the exogenous process $\{X_t\}$ models random fluctuations and it is assumed to follow a geometric Brownian motion:

$$dX_t = \mu X_t dt + \sigma X_t dW_t. \tag{2.2}$$

One may think at the stochastic process $\{X_t\}$ as the factor driving the market growth. In this sense, the parameter μ measures the expected instantaneous growth rate while the parameter σ is a measure of the riskiness of the investment.

Following Hugonnier and Morellec (2007), the utility a firm gets from the stream of cash-flow $\{\pi(X_t)\}$ is described by:[1]

$$V(x) = E_x \left[\int_0^\infty \mathrm{e}^{-\rho t} U\left(\pi(X_t)\right) dt \right],$$

being ρ the subjective discount rate common to both firms, and $U(\cdot)$ the instantaneous utility function. For analytical tractability, in this chapter, we restrict attention to the class HARA instantaneous utility functions, which are defined, for $x > 0$, by:

$$U_i(x) = \frac{x^{1-\gamma}}{1-\gamma},$$

where we restrict attention to $\gamma \in [0, 1)$ since it turns out that relevant

[1] $E_x\left[g(X_t)\right] = E\left[g(X_t) \mid X_0 = x\right]$.

optimal quantities are not defined for $\gamma \geq 1$.

In the sequel, we formulate and solve the decision problem faced by both firms. As it is standard in strategic real options game theory, throughout this chapter, we shall indicate the first firm investing in the market as the leader and the firm investing after the leader as the follower. We study two cases: in the former, the role each firm plays in the game is preassigned, which gives rise to a sequential game; in the latter, the role of each firm is part of the solution, which results in a preemption game. Both cases are solved backward in time by solving first the decision problem of the follower for a given capacity choice of the leader.

To take into account different risk-aversion configurations, we will consider three cases:

1. the symmetric case, where the risk aversion of market participants is the same ($\gamma_L = \gamma_F = \gamma > 0$);
2. the follower is risk-averse ($\gamma_F > 0$ and $\gamma_L = 0$);
3. the leader is risk-averse ($\gamma_L > 0$ and $\gamma_F = 0$).

2.2.1 Follower's strategy

The follower's combined entry-capacity choice problem does not depend on the type of game under consideration. Assume the leader entered the market with capacity Q_L and denote by Q_F the follower's investment size. At the random investing time T_F, the follower enters the market by exchanging the lump sum cQ_F, which represents the cost required to undertake the investment with the stochastic stream of cash flow $\{\pi_F(Q_F, X_t)\} = \{p\,(Q_F + Q_L, X_t)\,Q_F\}$. Therefore, the decision problem of the follower is:

$$V_F(x) = \max_{T_F, Q_F \geq 0} E_x[\int_{T_F}^{\infty} e^{-\rho t} U(Q_F p(Q_F + Q_L, X_t))dt] - U(cQ_F). \tag{2.3}$$

We use backward induction to solve the follower's problem. First, suppose that the firm invests as soon as the leader. For any initial level x of the random process $\{X_t\}$, we determine the optimal capacity $\widehat{Q}_F(x, Q_L) = \arg\max_{Q_F \geq 0} F(x, Q_F, Q_L)$, where:

$$F(x, Q_F, Q_L) := E_x[\int_0^\infty e^{-\rho t} U(Q_F X_t(1 - \eta(Q_F + Q_L)))dt] - U(cQ_F). \tag{2.4}$$

This gives the optimal capacity level in terms of a nonlinear implicit equation. After that, we derive the investment threshold using standard methods in real options. The following proposition gives the follower's optimal strategy:

Proposition 1. *Define parameters B and β as*

$$B_F = \frac{2}{(\gamma_F - 1)(2\mu - \gamma_F \sigma^2) + 2\rho} \tag{2.5}$$

$$\beta = \frac{\sigma^2 - 2\mu + \sqrt{(\sigma^2 - 2\mu)^2 + 8\rho\sigma^2}}{2\sigma^2}. \tag{2.6}$$

1. *The follower's optimal capacity when investing at level x, $\widehat{Q}_F(x, Q_L)$, solves:*

$$B_F x^{1-\gamma_F}(1 - 2\eta\widehat{Q}_F(x, Q_L) - \eta Q_L)(1 - \eta(\widehat{Q}_F(x, Q_L) + Q_L))^{-\gamma_F} = c^{1-\gamma_F}; \tag{2.7}$$

2. *The utility-maximizing investment strategy of the follower consists in the pair $\left(\widehat{Q}_F(Q_L), \widehat{T}_F(Q_L)\right)$, where:*

$$\widehat{T}_F(Q_L) = \inf_{t \geq 0} \left\{t : X_t \geq \widehat{X}_F(Q_L)\right\},$$

and

$$\widehat{X}_F(Q_L) = \frac{c(1 + \beta - \gamma_F)}{1 - \eta Q_L} \left(\frac{B_F(\beta + \gamma_F - 1)}{\beta^{\gamma_F}}\right)^{\frac{1}{\gamma_F - 1}}, \tag{2.8}$$

$$\widehat{Q}_F(Q_L) = \frac{(1 - \gamma_F)(1 - \eta Q_L)}{\eta(\beta - \gamma_F + 1)}. \tag{2.9}$$

3. The follower's value function is given by:

$$\widehat{V}_F(x, Q_L) = \begin{cases} A_F(Q_L)x^\beta \ \text{if } x < \widehat{X}_F(Q_L) \\ \widehat{F}(x, Q_L) \ \text{if } x \geq \widehat{X}_F(Q_L), \end{cases} \tag{2.10}$$

where $\widehat{F}(x, Q_L) = \max_{Q_F \geq 0} F(x, Q_F, Q_L)$ *and* $A_F(Q_L) = \frac{\widehat{F}(\widehat{X}_F(Q_L), Q_L)}{\widehat{X}_F(Q_L)^\beta}$.

The investment threshold and capacity level of follower under three cases are defined in B.1 and are the risk-averse generalization of the formulas obtained in Huisman and Kort (2015).

2.2.2 Leader's strategies

Having the advantage to be the first to enter the market, the leader solves the decision problem taking into account the strategy of the follower. Two possible alternatives are available to the leading firm, as the leader might prefer either to deter the follower's entry as much as possible or to let the follower enter as soon as the leader has invested. Adopting the terminology of Huisman and Kort (2015), we call entry deterrence strategy a pair (Q_L^D, X_L^D), consisting of the leader's capacity size and investment threshold, such that the leader invests strictly before the follower, that is $X_L^D < \widehat{X}_F(Q_L^D)$. Alternatively, an entry accommodation strategy is a pair (Q_L^A, X_L^A) such that the follower invests at the same time as the leader, which is $X_L^A = \widehat{X}_F(Q_L^A)$.

Leader's deterrence strategy

An entry deterrence strategy allows the leading firm to enjoy a period of monopoly, from the leader's time of entry until the follower invests. This sequential investment pattern generates the following utility maximization problem:

$$\begin{aligned} V_L^D(x) = \max_{T_L, Q_L \geq 0} E_x\Big[\int_{T_L}^{\infty} e^{-\rho t} U(Q_L p(Q_L, X_t)) dt\Big] - U(cQ_L) \\ + E_x\Big[\int_{\widehat{T}_F(Q_L)}^{\infty} e^{-\rho t} (U(Q_L p(\widehat{Q}_F(Q_L) + Q_L, X_t)) - U(Q_L p(Q_L, X_t))) dt\Big] \end{aligned} \tag{2.11}$$

The rationale of equation (2.11) is as follows. The leader enjoys a period of monopoly until time $\widehat{T}_F(Q_L) > T_L$ when the follower enters the market with capacity $\widehat{Q}_F(Q_L)$. At that time, the leader loses her monopolist position and is forced to exchange the profit flow $\{p(Q_L, X_t)Q_L\}$ with the profit flow $\{p(Q_L + \widehat{Q}_F(Q_L), X_t)Q_L\}$.

The problem is solved again with the help of backward induction. First, suppose that the leader invests at time $t = 0$. For any initial level x of the random shock, we determine the optimal capacity that maximizes the leader value under entry deterrence strategy, given by:

$$\begin{aligned} L^D(x, Q_L) = E_x[&\int_0^\infty e^{-\rho t}(U(Q_L p(Q_L, X_t))dt \\ &+ \int_{\widehat{T}_F(Q_L)}^\infty e^{-\rho t}(U(Q_L p(\widehat{Q}_F(Q_L) + Q_L, X_t)) - U(Q_L p(Q_L, X_t)))dt] \\ &- U(cQ_L)). \end{aligned} \tag{2.12}$$

Define $\widehat{Q}_L^D(x) = \arg\max_{Q_L \geq 0} L^D(x, Q_L)$. To generate an entry deterrence strategy, $\widehat{Q}_L^D(x)$ must be such that $\widehat{X}_F\left(\widehat{Q}_L^D(x)\right) < x$, otherwise the follower's entry threshold is at least equal to the current level of the state and this makes the follower enter immediately, thus preventing the strategy to be of entry deterrence type. Inverting the previous relation, an entry deterrence strategy is possible if $\widehat{Q}_L^D(x) > \bar{Q}(x)$, where:

$$\bar{Q}(x) = \frac{1}{\eta}\left(1 - \frac{c(\beta - \gamma_F + 1)}{x}\left(\frac{\beta^{\gamma_F}}{B_F(\beta + \gamma_F - 1)}\right)^{\frac{1}{1-\gamma_F}}\right). \tag{2.13}$$

Next proposition summarizes the relevant results for the leading firm under entry deterrence:

Proposition 2. *Consider the parameter β as defined in equation (2.6) and the parameter $B_L = \frac{2}{(\gamma_L - 1)(2\mu - \gamma_L\sigma^2) + 2\rho}$.*

1. *Entry deterrence is a feasible strategy whenever $x \in (x_1^D, x_2^D)$. Under entry deterrence, the optimal leader's capacity, $\widehat{Q}_L^D(x)$, implic-*

itly solves

$$\widehat{X}_F^{1-\beta-\gamma_L}(\widehat{Q}_L^D(x)) = \frac{(\gamma_L-1)(-\beta+\gamma_F-1)c^{-\gamma_L}x^{-\beta-\gamma_L}(\frac{\beta-\beta\eta\widehat{Q}_L^D(x)}{\beta-\gamma_F+1})^{\gamma_L}}{B_L(\gamma_L+(\beta+1)\eta\widehat{Q}_L^D(x)-\gamma_L\eta\widehat{Q}_L^D(x)-1)} \cdot \frac{B_L x c^{\gamma_L}(2\eta\widehat{Q}_L^D(x)-1)+cx^{\gamma_L}(1-\eta\widehat{Q}_L^D(x))^{\gamma_L}}{\left((\beta-\gamma_F+1)(\frac{\beta-\beta\eta\widehat{Q}_L^D(x)}{\beta-\gamma_F+1})^{\gamma_L}-\beta(1-\eta\widehat{Q}_L^D(x))^{\gamma_L}\right)}. \tag{2.14}$$

The cumbersome expression of x_1^D and x_2^D are relegated in B.2.

2. *The leader's utility-maximizing investment strategy under entry deterrence consists in the pair $(\widehat{Q}_L^D, \widehat{T}_L^D)$, where*

$$\widehat{T}_L^D = \inf_{t\geq 0}\left\{t : X_t \geq \widehat{X}_L^D\right\}$$

and

$$\widehat{X}_L^D = \frac{c(\beta-\gamma_L+1)}{\beta}\left(\frac{B_L(\beta+\gamma_L-1)}{\beta}\right)^{\frac{1}{\gamma_L-1}} \tag{2.15}$$

$$\widehat{Q}_L^D = \frac{1-\gamma_L}{\eta(\beta-\gamma_L+1)}. \tag{2.16}$$

3. *The value function of the leader under entry deterrence is*

$$\widehat{V}_L^D(x) = \begin{cases} A_L^D x^\beta \text{ if } x < \widehat{X}_L^D \\ \widehat{L}^D(x) \text{ if } x \geq \widehat{X}_L^D \end{cases} \tag{2.17}$$

where $\widehat{L}^D(x) = \max_{Q_L\geq 0} L^D(x, Q_L)$ and $A_L^D = \frac{\widehat{L}^D(\widehat{X}_L^D)}{(\widehat{X}_L^D)^\beta}$.

Corollary 1. *Consider the deterrence strategy under three cases. The corresponding equations for principal parameters are given in the table below:*

Parameters	$\gamma_L=\gamma_F=\gamma>0$	$\gamma_F>0,\gamma_L=0$	$\gamma_F=0,\gamma_L>0$
$\widehat{X}_L^D$	$\frac{c(\beta-\gamma+1)}{\beta}\left(\frac{B(\beta+\gamma-1)}{\beta}\right)^{\frac{1}{\gamma-1}}$	$\frac{c(\beta+1)}{B_L(\beta-1)}$	$\frac{c(\beta-\gamma_L+1)}{\beta}\left(\frac{B_L(\beta+\gamma_L-1)}{\beta}\right)^{\frac{1}{\gamma_L-1}}$
$\widehat{Q}_L^D$	$\frac{1-\gamma}{\eta(\beta-\gamma+1)}$	$\frac{1}{\eta(\beta+1)}$	$\frac{1-\gamma_L}{\eta(\beta-\gamma_L+1)}$
x_1^D	*eq. (B.10)*	*eq. (B.14)*	*eq. (B.16)*
x_2^D	*eq. (B.11)*	*eq. (B.15)*	*eq. (B.17)*

Leader's accommodation strategy

Under entry accommodation, the leader chooses her strategic variables so that the follower invests immediately after the leader. For an accommodation strategy to be feasible, it is necessary that $Q_L^A(x) \leq \bar{Q}(x)$, that is the capacity chosen by the leader must be such that $\widehat{X}_F(Q_L^A(x)) \geq x$. This requires a level of the random shock at least equal to x^A. Then, the follower finds it convenient to enter the market immediately. The leader's decision problem under accommodation strategy reads:

$$V_L^A(x) = E_x[\int_{T_L}^{\infty} e^{-\rho t}(U(Q_L p(Q_L + \widehat{Q}_F(x, Q_L), X_t))dt] - U(cQ_L)). \tag{2.18}$$

We again make use of backward induction. Suppose the leader invests at time $t = 0$. For any initial level x, we need to determine $\widehat{Q}_L^A(x) = \arg\max_{Q_L \geq 0} L^A(x, Q_L)$, where:

$$L^A(x, Q_L) = E_x[\int_0^{\infty} \mathrm{e}^{-\rho t}(U(p(Q_L + \widehat{Q}_F(x, Q_L), X_t)Q_L)dt] - U(rcQ_L)). \tag{2.19}$$

We note that the problem of finding $\widehat{Q}_L^A(x)$ has a complicated structure since the function $\widehat{Q}_F(x, Q_L)$ is itself an implicit function of Q_L and x.

Proposition 3. *Under entry accommodation, the optimal capacity of the leader when entry at the level of the random shock x, which we denote by $\widehat{Q}_L^A(x)$, and that of the follower are implicitly defined by (2.7) and*

$$\frac{(1 - \eta(Q_F(Q_L) + Q_L))^{1-\gamma_L}}{x^{\gamma_L - 1} c^{1-\gamma_L} B_L^{-1}} = \frac{\gamma_F\left(\eta\left(2Q_F(Q_L) + Q_L\right) - 1\right) - 2\left(1 - \eta(Q_F(Q_L) + Q_L)\right)}{\gamma_F\left(\eta\left(2Q_F(Q_L) + Q_L\right) - 1\right) - \eta\left(2Q_F(Q_L) + 3Q_L\right) + 2}. \tag{2.20}$$

The symmetric case in which $\gamma_F = \gamma_L = \gamma$ decouples the system. The following explicit function defines the follower's optimal capacity:

$$Q_F(x) = \Psi\left(Q_L^A(x)\right), \tag{2.21}$$

where $Q_L^A(x)$ solves the following expression:

$$\frac{(1-\eta(Q_F(Q_L)+Q_L))^{1-\gamma}}{x^{\gamma-1}c^{1-\gamma}B^{-1}} = \frac{\gamma\left(\eta\left(2Q_F(Q_L)+Q_L\right)-1\right)-2\left(1-\eta(Q_F(Q_L)+Q_L)\right)}{\gamma\left(\eta\left(2Q_F(Q_L)+Q_L\right)-1\right)-\eta\left(2Q_F(Q_L)+3Q_L\right)+2}. \tag{2.22}$$

The expression of the function $\Psi(Q)$ is relegated in B.3. The asymmetric case in which $\gamma_F > 0, \gamma_L = 0$, the optimal capacity of the follower at the same level of the random shock is defined by (B.26). The asymmetric case in which $\gamma_L > 0, \gamma_F = 0$, the optimal capacity of the follower at the same level of the random shock is defined by (B.29).

Proposition 3 reduces the problem of finding the optimal capacity of the leader under entry accommodation, which depends on the optimal capacity the follower chooses, into an implicit equation to be solved. From these results, the function $\widehat{Q}_L^A(x)$, together with the optimal value function $\widehat{V}^A(x)$, can be approximated straightforwardly. Details of the entry accommodation strategy of the leader see in B.3.

2.3 Strategic capacity choice under risk-aversion: Exogenous leadership

2.3.1 Symmetric case

The leader problem is solved as explained in Huisman and Kort (2015). The solution of the leader's problem, consisting of choosing whether to deter or to accommodate the follower's entry, depends on the relative location between x_2^D and x^A. Although in our setup it is not possible to compare the boundaries analytically, numerical evidence based on an extensive set of experiments shows that, in case of symmetric games, $x^A \leq x_2^D$. With this relationship, it is possible to determine the leader's optimal strategy, as follows. For $x \leq x^A$, only the deterrence strategy is feasible, while for $x \geq x_2^D$, entry accommodation is the only possibility. In the region (x^A, x_2^D), where both strategies are feasible, the leader chooses to have the highest possible value. To describe the leader's choice in that region, we make use of the function $M(x)$, defined by:

$$M(x) = \begin{cases} D & \text{if } \widehat{V}_L^D(x) \geq \widehat{V}_L^A(x) \\ A & \text{otherwise,} \end{cases}$$

which returns the leader's optimal strategy for each $x \in (x^A, x_2^D)$. We summarize the leader's equilibrium quantities in Table 2.1.

Region	Strategy	$\widehat{V}_L(x)$	$\widehat{X}_L$	$\widehat{Q}_L(x)$
$\left\{x \in (x_1^D, x^A]\right\}$	Deterrence	$\widehat{V}_L^D(x)$	$\max\left\{x, \widehat{X}_L^D\right\}$	$\widehat{Q}_L^D\left(\widehat{X}_L\right)$
$\left\{x \in (x^A, x_2^D]\right\}$	Both	$\widehat{V}_L^{M(x)}(x)$	$\max\left\{x, \widehat{X}_L^{M(x)}\right\}$	$\widehat{Q}_L^{M(x)}\left(\widehat{X}_L\right)$
$\left\{x \in (x_2^D, \infty)\right\}$	Accommodation	$\widehat{V}_L^A(x)$	$\max\left\{x, \widehat{X}_L^A\right\}$	$\widehat{Q}_L^A\left(\widehat{X}_L\right)$

Table 2.1: Summary of the relevant leader's equilibrium quantities under exogenous leadership. $\widehat{V}_L(x)$: equilibrium value function; $\widehat{X}_L$: equilibrium investment threshold; $\widehat{Q}_L(x)$: equilibrium capacity. The function $M(x)$ returns strategy D if the value of applying deterrence is higher than the value of applying accommodation, and vice versa.

c	ρ	μ	η
0.1	0.1	0.06	0.05

Table 2.2: Parameter values used in the analysis.

Unless otherwise stated, we use the set of parameters' values provided in Table 2.2.

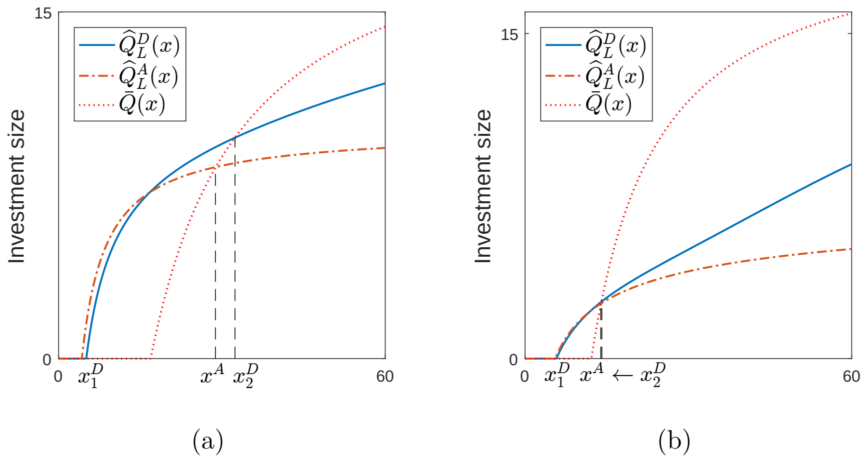


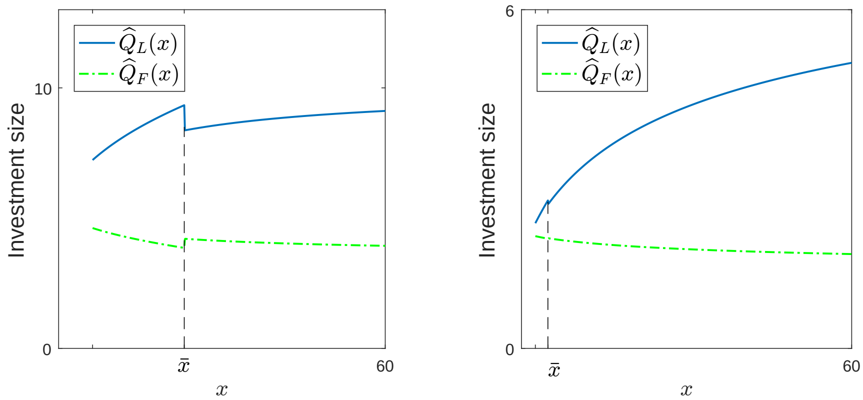
Figure 2.1: Leader's optimal capacity choice: deterrence versus accommodation strategy under risk-aversion. Entry deterrence is a feasible strategy for $x \in (x_1^D, x_2^D)$. Entry accommodation is a feasible strategy for $x \geq x^A$. Panel 2.1(a) shows a case of low risk-aversion ($\gamma = 0.1$); Panel 2.1(b) shows a case of high risk-aversion ($\gamma = 0.8$). The remaining parameter values are those in Table 2.2.

Figure 2.1 presents two examples of the functions $\widehat{Q}_L^D(\cdot)$ and $\widehat{Q}_L^A(\cdot)$ with different levels of risk-aversion. The figure highlights the typical effects of risk-aversion on the optimal strategies available to the leader. First, increased risk-aversion reduces the optimal investment scale. As risk-aversion increases, so does the leader's willingness to protect the business against markets' breakdown. This effect creates an incentive for the leader to reduce the amount invested. However, the example of Figure 2.1 also shows an apparently unexpected effect. As risk-aversion increases, the difference, in terms of optimal investment size, between deterrence and accommodation strategy tends to disappear. Put differently, in a market characterized by two highly risk-averse firms, the optimal investment size of the non-preemptive leader does not depend on whether she chooses to accommodate or deter investment of the follower. This effect is clearly visible when comparing panels 2.1(a) and 2.1(b). An increase of risk-aversion reduces both x_2^D and x^A, the reduction of x^A occurring at a higher speed. As γ approaches to 1, both boundaries assume the same value. This enlarges the accommodation region and, at the same time, reduces the size of the deterrence region.

More importantly, as risk-aversion increases, $\widehat{Q}_L^A(x)$ approaches $\widehat{Q}_L^D(x)$. This effect is more pronounced in the region in which deterrence is feasible. Economic intuition suggests that two forces cause this phenomenon. First, independently on the strategy used by the leader, an increase in risk-aversion causes the follower to invest earlier.[2] This means the period of monopoly the leader enjoys is exogenously reduced when risk-aversion increases, thus reducing the appealing of a deterrence strategy. Second, a deterrence strategy implies investing in a significant production plant to delay the follower's entry into the market. This undoubtedly increases the investment risk profile. As the leader becomes more risk-averse, the incentive to make a large investment is less pronounced. This, in turn, makes the follower invest earlier, thus reducing the leader's extra profits due to a temporary monopoly. In the end, when risk-aversion is sufficiently high, entry deterrence and entry accommodation become indistinguishable in terms of the size of the investment.

High risk-aversion reduces the leader's incentive to over-invest in order to delay follower's entry (compare, for instance, both panels of Figure 2.2). By over-investing in the deterrence region, the leader obtains a gain in utility due to the delay of the follower's entry. On the other hand, since entry costs are sunk, the risk of facing investment losses due to unfavorable future market conditions increases with the size of the investment. Thus over-investment also causes a loss in the leader's utility. As risk-aversion increases the latter effect becomes predomi-

[2] Chronopoulos et al. (2013), the first to document this effect, show that the more risk-averse the monopolist, the lower the investment size. Due to the increased concern about risk, the monopolist adjusts the risk profile of the investment by investing less. This reduces the value of the option to wait since a lower amount invested requires a lower level of market size to maximize the investment's profitability.



(a) (b)

Figure 2.2: Equilibrium capacity choice when firms' roles are preassigned: effects of risk-aversion. Panel 2.2(a) shows a case of low risk-aversion ($\gamma = 0.1$); Panel 2.2(b) shows a case of high risk-aversion ($\gamma = 0.8$). In both panels, entry deterrence is applied for $x < \bar{x}$ while entry accommodation is applied for $x \geq \bar{x}$. The remaining parameter values are those in Table 2.2.

nant, thus making the leader less willing to delay her competitor's entry. In terms of investment timing, this phenomenon implies that increased risk-aversion reduces the values of the leader's option value to wait. By investing in a smaller production plant, the leader requires a lower level of market size to maximizes the profitability of her investment. This is shown in Figure 2.3. From that figure, it is also evident that the period of time in which the leader benefits from a temporary monopoly position (which can be measured, in Figure 2.3, by intersecting vertical lines with the relevant curves) decreases with risk-aversion.

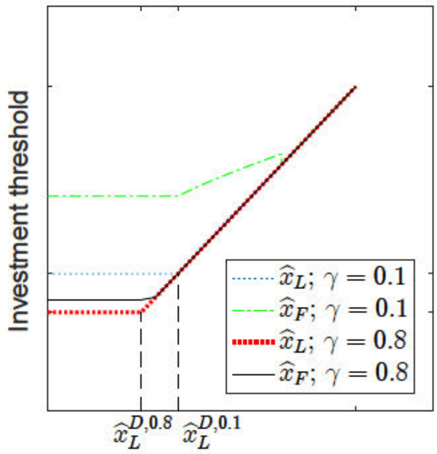


Figure 2.3: Leader's and follower's investment threshold plotted against the level of random shock when firms' roles are a priori assigned. The figure compares a scenario with low risk-aversion ($\gamma = 0.1$) and a scenario with high risk-aversion ($\gamma = 0.8$). The symbol $x_L^{D,\gamma}$ is used to distinguish between the two levels of risk-aversion. The remaining parameter values are those in Table 2.2.

2.3.2 Asymmetric case: the follower is risk-averse

In this section, we will consider the case when the leader is risk-neutral ($\gamma_L = 0$), and the follower is risk-averse. Figure 2.4 shows an example function $\widehat{Q}_L^D(\cdot)$ and $\widehat{Q}_L^A(\cdot)$ for the case when γ_F is low. We observe that in this case $x_2^D < x^A$, which changes the relevant leader's equilibrium quantities (see Table 2.3). As a consequence, there is a region in which neither deterrence strategy nor accommodation strategy can be implemented; therefore, the leader does not make any investments.

Region	Strategy	$\widehat{V}_L(x)$	$\widehat{X}_L$	$\widehat{Q}_L(x)$
$\left\{x \in (x_1^D, x_2^D]\right\}$	Deterrence	$\widehat{V}_L^D(x)$	$\max\left\{x, \widehat{X}_L^D\right\}$	$\widehat{Q}_L^D\left(\widehat{X}_L\right)$
$\left\{x \in (x_2^D, x^A)\right\}$	No invest	-	-	-
$\left\{x \in [x^A, \infty)\right\}$	Accommodation	$\widehat{V}_L^A(x)$	$\max\left\{x, \widehat{X}_L^A\right\}$	$\widehat{Q}_L^A\left(\widehat{X}_L\right)$

Table 2.3: Summary of the relevant leader's equilibrium quantities under exogenous leadership if the follower is risk-averse.

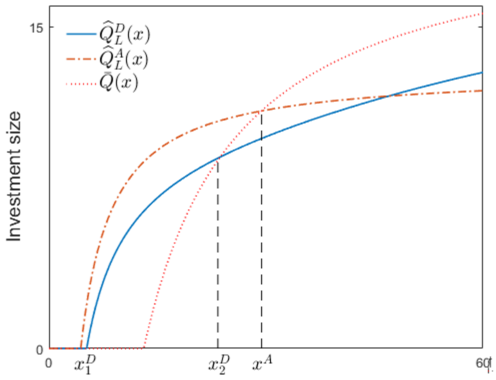


Figure 2.4: Leader's optimal capacity choice: case of low risk-aversion ($\gamma_F = 0.1$).

With increasing risk-aversion, the follower invests earlier, and his optimal scale of investment decreases. If the follower will tend to be more risk-averse ($\gamma_F = 0.7$), the possibility of implementing a deterrent strategy disappears (see Fig. 2.5). When γ_F approaching 1, the deterrence region decreases drastically. Increasing risk aversion by the leader decreases both x_2^D and x^A. In a market that characterizes a follower with a high degree of risk-aversion, the leader does not have the opportunity to take advantage of the monopoly period. Over-investing costs are very high, which makes it impossible to implement a deterrence strategy. High risk-aversion of the follower increase the leader's incentives to under-invest.

As risk aversion increases, over-investing leads to a loss in the leader's utility, making the leader less likely to delay the entry of a competitor. So with $\gamma_F = 0.7$, this effect becomes predominant, and the leader altogether does not delay the entry of the follower into the market. If increase risk-aversion of the followers, it reduces the values of the leader's option value to wait.

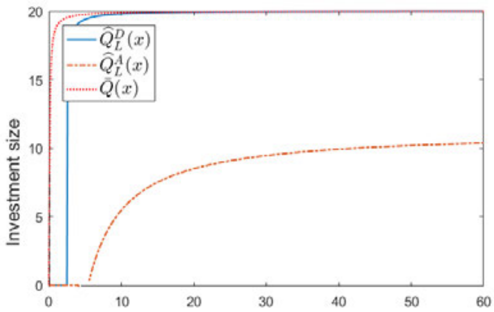
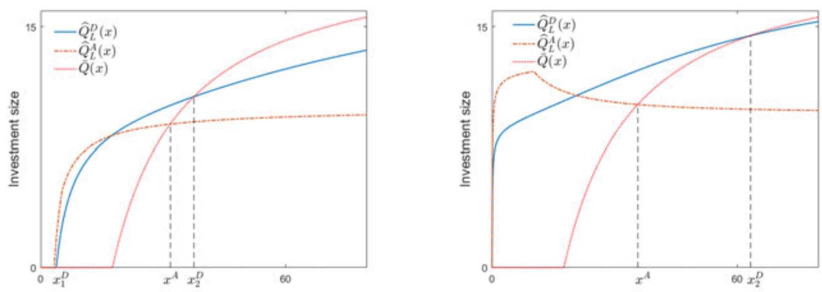


Figure 2.5: Leader's optimal capacity choice: case of high risk-aversion ($\gamma_F = 0.7$).

2.3.3 Asymmetric case: the leader is risk-averse

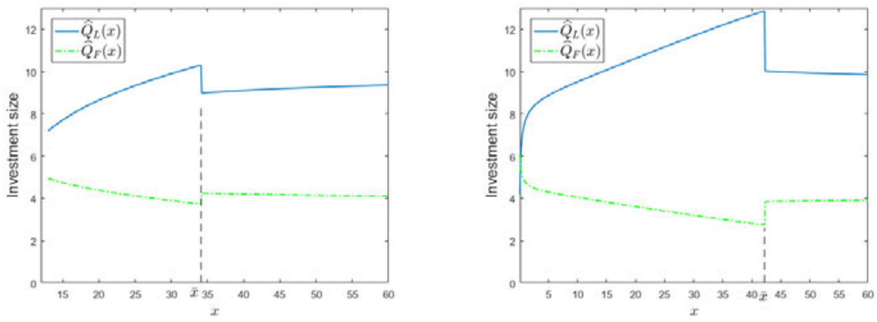
In this section, we consider the case opposite to the one described above, when the follower is risk-neutral ($\gamma_F = 0$). Figure 2.6 presents two examples of the functions $\widehat{Q}_L^D(\cdot)$ and $\widehat{Q}_L^A(\cdot)$ with different levels of risk-aversion by a leader. In the case when the follower is risk-neutral, then with an increase in risk aversion the leader increases his optimal investment scale. As the leader increases risk aversion, the difference, from the point of view of the optimal investment size, between deterrence and accommodation strategies tends to increase sharply. When γ_L approaching 1, the deterrence region increases. Increasing risk aversion by the leader increases both x_2^D and x^A. It is worth noting that an increase in x_2^D occurs at a faster rate than x^A.



(a) (b)

Figure 2.6: Leader's optimal capacity choice: under different levels of risk-averse. Panel 2.6(a) shows a case of low risk-aversion ($\gamma_L = 0.1$); Panel 2.6(b) shows a case of high risk-aversion ($\gamma_L = 0.7$).

With increasing risk aversion, $\widehat{Q}_L^A(\cdot)$ is moving away from $\widehat{Q}_L^D(\cdot)$, especially in a region where a deterrence strategy is feasible. Once again, we observe that risk-aversion increases, investing earlier. We can observe if the leader begins to be risk-averse ($\gamma_L = 0.7$), the duration of his monopolistic period increases.



(a) (b)

Figure 2.7: Equilibrium capacity choice when firms' roles are preassigned: effects of leader's risk-aversion. Panel 2.7(a) shows a case of low risk-aversion ($\gamma_L = 0.1$); Panel 2.7(b) shows a case of high risk-aversion ($\gamma_L = 0.7$). In both panels, entry deterrence is applied for $x < \bar{x}$ while entry accommodation is applied for $x \geq \bar{x}$.

High risk-aversion increases the leader's incentive for over-invest in order to delay follower's entry (compare both panels of Figure 2.7). The leader with high risk-aversion ($\gamma_L = 0.7$) obtains a gain in utility due to the delay of the follower's entry. By over-investment causes a benefit in the leader's utility. As a leader's risk-aversion increases, this effect becomes predominant, thus making the leader more willing to delay her competitor's entry.

2.4 Strategic capacity choice under risk-aversion: Endogenous leadership

In this section, we analyse the case in which the role each firm plays in the game is not given ax-ante. In this situation, each firm should decide whether to become the leader and get the value function $\widehat{V}_L(x)$ in Table 2.1, or the follower, and get the value function $\widehat{V}_F(x, Q_L)$ in (2.10). Panel 2.8(a) gives a graphical representation of the game at hand, which posses the features of a preemption game: each firm is willing to become the leader, for there is a first-mover advantage due to the period of temporary monopoly the leader enjoys. Preemption games in continuous time are studied in the seminal paper of Fudenberg and Tirole (1985) in the deterministic case and extended to the stochastic case in Thijssen et al. (2012). According to this literature, the principle of rent equalization must be used to solve the game. Each firm, for fear to be preempted, is willing to invest just before its rival. The incentive to preempt each other vanishes at the so-called preemption thresholds $\widehat{X}_P$, defined by:

$$\widehat{V}_L\left(\widehat{X}_P\right) = \widehat{V}_F\left(\widehat{X}_P, \widehat{Q}_L\left(\widehat{X}_P\right)\right). \tag{2.23}$$

At the preemption threshold, the value of being the leader is equal to the value of being the follower. Any further attempt to invest before this threshold makes the leader's value lower than the follower's value, and the leader's advantage vanishes. In mixed strategies, with the same probability, one of the two firms enters the market exactly when the process X_t touches the preemption threshold, with capacity $\widehat{Q}_P = \widehat{Q}_L\left(\widehat{X}_P\right)$. The rival assumes the role of the follower, entering the market at the threshold $\widehat{X}_{F,P} = \widehat{X}_F\left(\widehat{Q}_P\right)$, with capacity $\widehat{Q}_{F,P} = \widehat{Q}_F\left(\widehat{Q}_P\right)$.

2.4.1 Symmetric case

Panel 2.8(b) illustrates the effects of risk-aversion on the equilibrium investment thresholds when firms' roles are endogenously determined. The panel signals the dramatic reduction of the first-mover advantage in the preemption game as risk-aversion increases. This effect is particularly pronounced at the highest admissible values of risk-aversion, as for those value both firms invest almost at the same time. The economic explanation of this reduction lies in the expression of the follower's entry threshold, (2.8). The preemption threshold, which defines the leader's optimal investment time, is slightly increasing in risk-aversion, thus signalling a light increment of the option value to wait for the leader's point of view. However the leader's optimal capacity is a decreasing function of risk-aversion. Indeed, increased risk-aversion makes the leader willing to lower the capacity of the production plant in order to reduce investment riskiness. This, in turn, eliminates the possibility, for the leader side, to over-invest to delay the follower investment.[3]

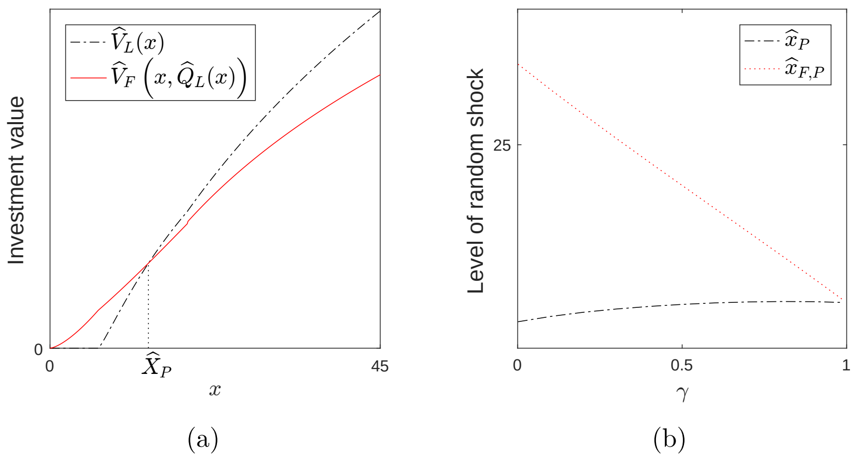


Figure 2.8: Panel 2.8(a) shows the value functions for the leader and the follower when firms' roles are endogenously determined. The risk-aversion parameter is set to $\gamma = 0.75(\sigma = 0.2)$; Panel 2.8(b) plots leader's and follower's investment threshold as a function of γ ($\sigma = 0.2$). The remaining parameter values are those in Table 2.2.

Next, we analyse the effects of risk-aversion in the equilibrium capacity size. Our main concern here is to understand whether one firm invests in a capacity size larger than its rival. Figure 2.9 displays four possible patterns, depending on the particular risk-return profile of the

[3]Due to space limitation, we present only one example of this phenomenon. However, we have run an extensive set of numerical experiments, all confirming the robustness of this result concerning changes in value of different parameters.

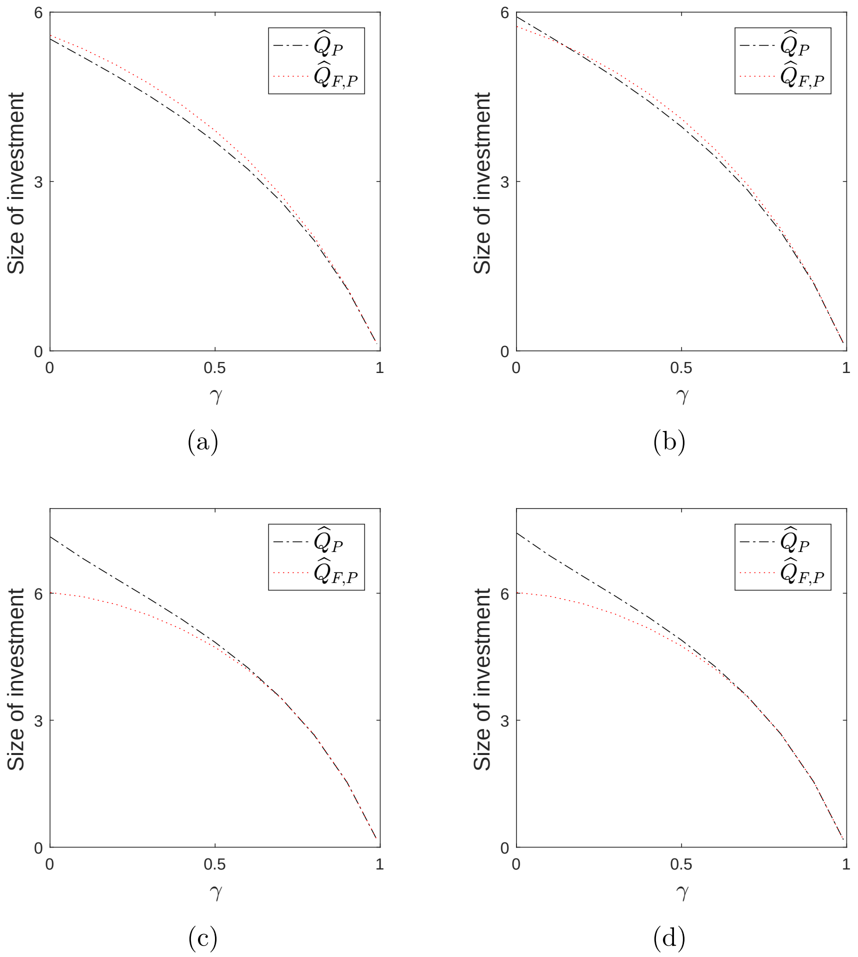


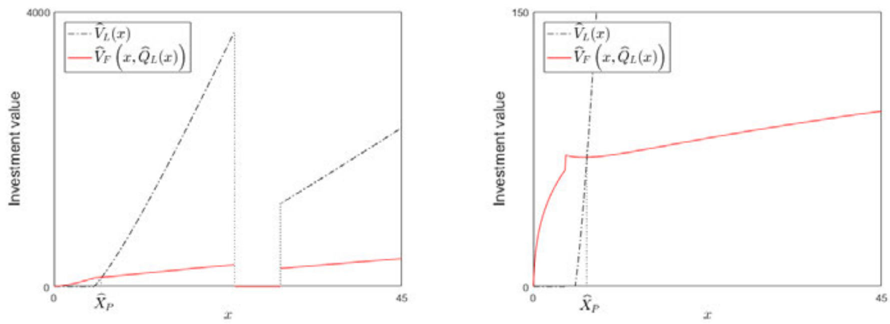
Figure 2.9: Equilibrium capacity sizes as a function of risk-aversion: endogenous firms' role. Panel 2.9(a) $\gamma = 0.1$ $\mu = 0.06$; Panel 2.9(b) $\gamma = 0.2$ $\mu = 0.06$; Panel 2.9(c) $\gamma = 0.1$ $\mu = 0.09$. Panel 2.9(d) $\gamma = 0.2$ $\mu = 0.09$; The remaining parameter values are those in Table 2.2.

investment. All panels present a common feature: when risk-aversion is very high, both firms invest, in equilibrium, in the same capacity size. However, for low to moderate levels of risk-aversion, the risk-return profile of the investment matters. Panels 2.9(a) and 2.9(b) present a situation where the expected instantaneous growth rate is relatively low ($\mu = 0.06$). In such situations, the leader's willingness to protect the business is reflected by the fact that in almost all the case the follower equilibrium capacity size is larger than that of the leader. Particularly interesting is the change of the leader's qualitative behaviour from very low to moderate levels of risk-aversion when the investment risk is high (panel 2.9(b)). A leader characterized by a low concern about risk-aversion is willing to over-invest as much as possible in order to delay its rival's entry, thus enjoying a longer period of monopoly. This makes the leader the largest firm in the market. However, as risk-aversion increases, the leader's fear of future market crashes prevails, and the effect is reversed. Panels 2.9(c) and 2.9(d) present a situation where the expected instantaneous market growth rate is relatively high. Here, for low to moderate levels of risk-aversion, the incentive to over-invest is strong enough to prevail, no matters the riskiness of the investment, and the leader is always the largest firm in the market.

2.4.2 Asymmetric case: the follower is risk-averse

As in paragraph 2.3.2, we also have the leader who is risk-neutral. There is a period of "No invest," in connection with which we get a graphic in the form of piecewise. We observe the value of the preemption threshold increase increases with the follower's level of risk-aversion. The first thing we observe when comparing two panels 2.10(a) and 2.10(b) is a significant reduction in investment value.

Investment is not profitable for small values of the random shock x. Then no firm wants to invest first, which is why the follower curve lies above the leader curve. For larger values than the preemption thresholds $\widehat{X}_P$, each firm wants to be the first investor. When γ_F approaching 1, the preemption thresholds $\widehat{X}_P$ slightly increases. Increased risk-aversion γ_F forces the leader to reduce the capacity of the production plant in

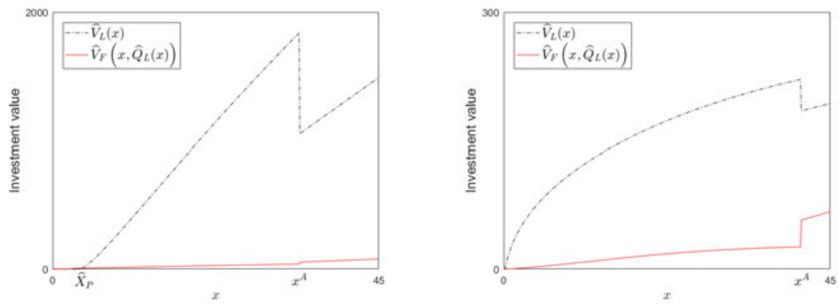


(a) (b)

Figure 2.10: The figure shows the value functions for the leader and the follower when firms' roles are endogenously determined. The risk-aversion parameter (γ_F) is set to 0.1 and 0.7 at panels 2.10(a) and 2.10(b), respectively.

order to reduce the riskiness of investments. When passing a certain level of γ_F, a leader is willing to over-invest as much as possible, in order to delay its rival's entry.

2.4.3 Asymmetric case: the leader is risk-averse



(a) (b)

Figure 2.11: The figure shows the value functions for the leader and the follower when firms' roles are endogenously determined. The risk-aversion parameter (γ_L) is set to 0.1 and 0.6 at panels 2.11(a) and 2.11(b), respectively.

As in paragraph 2.3.3, we also have the follower is risk-neutral. Recall that the dashed curve corresponds to the outcome if the entrant takes the leader position, where the pay-off of immediate investment is depicted. If the firm takes the position of the follower, one arrives at the solid curve. As in the previous paragraph, we observe a reduction in investment value is observed with increasing risk aversion.

When γ_L approaching 1, the preemption thresholds $\widehat{X}_P$ drastically decreases. With $\gamma_F = 0$, there is a significant permanent advantage to the second player. Increased risk aversion by the leader above a certain level causes the leader not to make investments. As the leader's risk-aversion increases, the leader's fear of future market crashes prevails. This fear makes the leader the smallest firm in the market. The follower capacity is always bigger than the leader capacity. The follower is delayed before x^A and invests immediately for $x \geqslant x^A$.

2.5 Conclusion

In this chapter, we discussed the joint role of uncertainty, competition, and risk-aversion in determining the equilibrium capacities and investment timing of firms in a symmetric duopoly. Despite its simplicity, our setup produces a set of novel predictions. Nevertheless, due to space limitations and tractability issues, the present chapter does not explore several aspects that deserve more attention.

The first limitation of the present chapter is that we do not pursue issues due to market incompleteness. We believe that introducing idiosyncratic risk and risk-aversion in a capacity-setting real options game along the lines of Hugonnier and Morellec (2017), Bensoussan et al. (2010), Henderson (2007), Hugonnier and Morellec (2007) is a promising avenue for future research. Also, we believe that analysing the role of incomplete information (see, for instance, Lambrecht and Perraudin, 2003) in a real options game with endogenous capacity is an important aspect that needs to be studied.

Chapter 3

Swing option valuation in literature: Research focus, trends and design

3.1 Introduction

In energy industries, exist many derivatives which are instruments to manage and reduce risk. The need for these tools is currently due to the multidimensionality of risks, restrictions of various nature (including technological), difficulties associated with electricity storage, and electricity price instability (for example, jumps, spikes, volatility, and seasonality). Traditional forward contracts often have some degree of flexibility, but this is still insufficient given the specifics' electricity market. The derivatives that have allowed the most flexibility have been known as "swing options" (also known as "swing contracts," "take-and-pay options," or "variable base-load factor contracts"). With the deregulation of energy markets, there is an increased interest in understanding and assessing the value of the optionality inherent in these contracts.

A standard swing contract is an agreement between a supplier and a buyer for a daily supply of a variable amount of electricity. This volume must be within a fixed range of allowable rates at a defined set of contract prices for a particular period of time. There are various possible additions to this typical contact; the market crises cause these additions to the standard version of the swing option.

Several published reviews explicitly demonstrate some models and approaches to swing options evaluation design (see, Løland and Lindqvist (2008), Lempa (2014) and Aïd (2015)). These papers, however, provide an incomplete overview, limiting their sample to a few selected studies. Moreover, these reviews lack a significant number of articles published in recent years.

Therefore in order to provide a full image of current studies and trends, the purpose of this chapter is to provide a more in-depth analysis of the scientific papers dealing with swing options. The chapter aims to review the scientific literature dealing with swing options, general outline directions for research and developments in this field, provide a detailed overview of the design of price models and numerical methods used, and characterize cutting-edge research areas.

The chapter is structured as follows. A brief description of the theoretical basis follows this introductory part, after which the research methodology is described. At the end of the chapter, we conclude with a summary of the main findings. The appendix contains a summary table of the main characteristics of the reviewed documents.

3.2 Theoretic context

Volumetric risk, i.e. the risk of a discrepancy between the predicted and actual volumes, together with the property of non-storage of electricity, necessitates the use of specific insurance instruments. These instruments include the swing contracts discussed below.

Considering article of Jaillet et al. (2004) as fundamental, it is worth noting the definition from this work:

> A swing contract is often bundled together with a standard base-load forward contract that specifies, for a given period and a predetermined price, the amount of the commodity to be delivered over that period.

So we can say the swing option is right to receive (or purchase) a given volume of energy. The buyer has the right to buy every day the amount of gas in the range from min daily contract quantity (DCQ) to the maximum DCQ at a given execution price K. However, it is necessary globally (for example, within a year) cumulative amount is in the range from the minimum annual contract quantity (ACQ) to maxima ACQ. It is worth noting that the strike price can be fixed, variable, or random. In general, swing options are often seen as multiple exercise American options. Without a penalty for overall consumption, the swing will be exercised in a "bang-bang" fashion (see Jaillet et al., 2004); that is, the lowest or highest limit of the permitted local constraint is met. If the number of rights (N) is equal to the number of exercise dates (n), the swing option's value is equal to the value of a strip of European options. To facilitate future discussion, we describe the most common characteristics of a swing option, including the following aspects:

- Delivery period (L): duration of one or more contract years
- Exercise time (J): can be exercised monthly ($J = 12$), weekly ($J = 52$) or daily ($J = 365$)
- Risk-free rate (r)

- Contractual price ($K_{i,j}$): Contracting parties can agree either on a fixed price or on an indexed price. In real contracts, the strike price is set based on the indexation principle under which the strike price is called the index.
- Gas (or electricity) price ($S_{i,j}$): $S_n = e^{-r\Delta t}F^S(t_n; t_{n+1})$
- Quantity ($q_{t_{i,j}}$):

  $mDCQ \leqslant q_{t_{i,j}} \leqslant MDCQ$, where DCQ is daily contract quantity
- Consumption policy ($\mathbf{q} = \{q_{t_{i,j}}\}$)
- Cumulative amount ($Q_{t_{i,j}}$): $Q_{t_{i,j}} = \sum_{k=1}^{j-1} q_{t_{i,k}}$

  $mACQ \leqslant Q_T \leqslant MACQ$, where ACQ is annual contract quantity
- Payoff: $q_{t_{i,j}}(S_{i,j} - K_{i,j})$
- Penalty: $\eta K_{i,J} \min\{Q_{T_i} - mACQ_i, 0\}$, where $\eta \in [0;1]$ is the penalty coefficient
- Make-up bank (M_i):

  $M_{i+1} = (M_i - m_i) + \max\{mACQ_i - c_i - Q_{T_i}; 0\}$
  MRL_i - the make-up bank recovery limit
  m_i - the usage of gas in the make-up bank in year i
- Carry-forward bank (C_i):

  $C_{i+1} = (C_i - c_i) + \max\{Q_{T_i} - \max\{mACQ_i + m_i; CB_i\}\}$
  CRL_i - the carry-forward bank recovery limit
  CB_i - the carry-forward base
  c_i - the usage of gas in the carry-forward bank in year i

An essential part of the swing option valuation is identifying sources of uncertainty and modelling their possible development. Among the main sources of uncertainty are electricity or fuel prices. To solve this problem, exist a wide variety of techniques can be applied. However, researchers most often use stochastic modelling, including geometric Brownian motion (GBM) or mean-reverting processes (MR). Certain specific types of uncertainty require specific models. We will highlight six main pricing models: Black and Scholes (BS), mean-reverting processes, mean-reversion with jumps (MRJ), regime-switching (RS), Lévy, and geometric Brownian motion.

This chapter draws attention to the widely used methods that can be found in the reviewed literature. There are usually five main formulations:

- Dynamic programming (DP)
- Variational inequality (VI)
- Partial differential equation (PDE)
- Partial integro-differential equation (PIDE)
- Backward stochastic differential equation (BSDE)

The valuation of swing options other than modelling the underlying electricity (or gas) pricing process involves solving the complex early exercise problem. However, these two aspects are, in principle, independent of each other.

The numerical method used in early swing options work is trees or lattices (TR). The Monte Carlo regression (MCR) is also the prevalent numerical method for pricing swing options, as proposed by Longstaff and Schwartz (2001) (originally used for Bermuda options). Not less common are the finite difference (FD) and the finite element (FE). Other numerical methods that researchers use in their work neural network (NN), quantisation (Q), and Malliavin calculus (Mal), either alone or in addition to the above methods.

3.2.1 Methodology

Most of the literature focuses on an overview of swing options. However, it is also worth noting that articles about hydroelectric reservoir also fall under this category, and only some articles are included about gas storage and multiple stopping-time problems.

This analysis incorporates the strengths of previously published literature reviews in this area. Initial searches in the SCOPUS database were limited to the following keyword phrase: "swing option." With a language restriction, the query returned one hundred and two results. Eighteen articles were excluded based on an abstract scan, resulting in eighty-four candidates selected for the review. Backtracking included link analysis from selected articles and case reviews. In total, 92 articles were selected for further study.

The review lacks studies that swing option pricing problem formulated as a bilevel decision problem, such as Pflug and Broussev (2009), Vayanos et al. (2011), Kovacevic and Pflug (2013), Kovacevic and Pflug (2014), Gross and Pflug (2016) and O'Malley et al. (2019). Besides, research related to game theory is excluded; for example, Iron and Kifer (2011) and Dolinsky et al. (2011) where introduced the concept of the multi-stopping Dynkin game.

The detailed results of this analysis are presented in a table in Appendix C. The credibility of the study was ensured only by reviewing academic articles from indexed scientific journals, conference proceedings and textbooks. The following section summarizes and discusses the key findings.

3.3 Valuation of commodity-based swing options

The first papers in the literature studying swing contracts date back to the nineties (see, for instance, Thompson (1995), Barbieri and Garman (1996), Pilipovic and Wengler (1998)). For example, Thompson (1995) applies the lattice-based method extended from Hull and White (1993) to determine the optimal exercise strategy for path-dependent conditional requirements. The work had its shortcomings, among which it is only provided for two special types of take-or-pay contracts; later, the work was expanded by other authors.

Before considering the current study results, we provide a brief critical summary of reviews of scientific papers on the valuation of swing options and discuss their observations. One of the earliest reviews of swing option pricing is a 2008 review by Løland and Lindqvist that summarizes existing research, divided into simulation and non-simulation based swing option valuation. The findings only confirm the attractiveness of swing options and the considerable scope for further research with make-up right and carry forward.

Later, in 2014, Lempa performed a fairly extensive survey of mathematical theories and techniques used to study swing options. This review focuses entirely on the martingale and Markovian methods. The complete review of energy derivatives was carried out in 2015 by Aïd, including storage and swings. There are 32 articles in comparison, which are included in our list with some exceptions. The paper presents observations on general thematic trends.

As we can see, the existing reviews, while providing valuable information on the subject, give only a fragmentary picture of approaches to the valuation of swing options, and their volume is limited to a small number of studies. This review attempts to build on predecessors' strengths and expand the sample size to advance research.

The earliest article mentioned in our summary table is Lari-Lavassani et al. (2001), where a discrete forest methodology is developed for swing options as a dynamically coupled system of European options. The swing option valuation is focused on the swing component. The swing part can be considered as a multi-exercise American-style option giving the holder multiple opportunities to exercise the swing actions. The valuation algorithms of the swing option are generalizations of those used for pricing American-style options. The lattice tree methods belong to a type of efficient algorithms for swing option valuation. For example, Clewlow et al. (2001) discuss optimal exercise decisions with a trinomial tree method.

The contribution describing in detail the valuation of standard swing options is Jaillet et al. (2004), where the authors develop a framework based on the dynamic programming with a trinomial tree. They use a well-known technique called "backward induction" to solve the pricing problem. This method starts from the contract's endpoint and works backwards in time by considering a mean-reverting spot price model. The proof of the convergence of the procedure can be found in (Lari-Lavassani et al., 2001), mentioned above. Keppo (2004) studies the hedging of a swing option using this model under a liquid forward contract and European call options market. In Barrera-Esteve et al. (2006), the authors propose and summarize several methods to evaluate swing options with penalty using both simulations and dynamic programming techniques. Bardou et al. (2009) use the so-called optimal quantization method to price swing options with the spot price following a mean-reverting process.

Various publications begin discussing of the "bang-bang" strategy (i.e., an all-or-nothing clause), where only the minimum or maximum consumption allowed by all the constraints is selected on each delivery day. Keppo (2004) proves that the optimal exercise policy in the case of the no-load penalty is bang-bang and derives explicit hedging strategies involving standard derivatives such as forwards and vanilla options. Among the early works, two articles are worth mentioning. Ross and Zhu (2008) show that the optimal strategy is the bang-bang style, and Bardou et al. (2010) got a similar result. However, these two papers are working with swing options that only allow local volume and global volume constraints without constraints on the number of exercise rights.

Wahab et al. (2010) e Wahab and Lee (2011) use the dynamic programming principle and take the maximum of two values when pricing an option at every time instance. It is worth mentioning that they do not show that the "bang-bang" strategy maximizes the profit of a holder of a swing option. For the discussion of the bang-bang consumption in swing options, we refer interested readers to Barrera-Esteve et al. (2006), Becker (2010) and Edoli et al. (2013).

Hinz (2006) is the scientist dealing with classical no-arbitrage models in the energy market price modelling, suggesting a framework where a flow commodity market converts into a market consisting of zero bonds and some additional risky asset. The model implementation points out how the agent's forecasting demand level plays an essential role in the equilibrium price definition.

Since 2008 there has been an increase in research into swing options. Swing options have become widespread, although pricing standard swing options are not trivial and even complicated with non-linear pay-offs and uncertain volumes. Breslin et al. (2008a) described and clarified several specific features of a standard gas swing contract. In their model, the volatility is a deterministic function of both the current time and the time-to-maturity. However, there is ample evidence that volatility in the gas markets is stochastic. Breslin et al. (2008b) discuss the risks and hedging of swing contracts.

Developments in the application of the tree method are two papers from 2009, Haarbrücker and Kuhn (2009) and Steinbach and Vollbrecht (2009). Haarbrücker and Kuhn advanced the valuation of swing options with ramping constraints on a scenario tree. Steinbach and Vollbrecht suggested the valuation technique by reducing a scenario tree and using a scenario fan. At the same time, Geman and Kourouvakalis (2008) proposed a method based on the quantization tree method.

Monte Carlo methods have been developed to price swing options and are one of the most common methods. These mainly include regression-based approaches Dörr (2003), duality methods Meinshausen and Hambly (2004), and those that parameterize the set of exercise level curves Ibánez (2004).

Another method used to evaluate the swing option is the finite element method, based on which the corresponding algorithm was developed in the article by Wilhelm and Winter (2008). When comparing their results with those obtained using the lattice method and Monte Carlo simulations, it was concluded that the finite element method showed promising results.

The impulse control was applied to the pricing of a multiple exercise American option, the pricing of swing options on the commodity market, and an investigation of sale strategies in an illiquid market. A limited amount of work used impulse control; see Dahlgren and Korn (2005) and Bernhart et al. (2012).

Barrera-Esteve et al. (2006) describing the stochastic control problem, where the delivery strategy is also analysed using neural networks. It is worth noting that this article uses forward contracts with different maturities. Also, documents on valuing swing options by DP include Baldick et al. (2006), where they construct a structural model in which the spot price of electricity is determined by supply and demand.

Carmona and Touzi (2008) formulated the valuation of swing options estimation as an optimal multiple-stopping problem, reduced to a compound single stopping problem. Moreover, they analysed this problem within a continuous-time Black-Scholes market and introduced the Malliavin calculus based Monte Carlo algorithm. Based on the previous article, Carmona and Dayanik (2008) examine the same problem and propose approximation formulas for more one-dimensional time-homogeneous diffusions.

The viscosity solution approach has been used for pricing swings in Basei et al. (2014) and Latifa et al. (2015). Indeed, Basei et al. only discusses the value function's characterization as a viscosity solution of a Hamilton-Jacobi-Bellman (HJM) equation with suitable boundary conditions in this framework. Instead, Latifa et al. generalize the results of Carmona and Touzi (2008) when the price process is allowed to jump and solved the optimal multiple stopping problem as the unique viscosity solution of HJM variational inequality.

Numerical partial differential equations e partial integro-differential equations approaches can give rise to more accurate prices, but they are mathematically complex. Consequently, we can see that the numerical resolution of the PDE approach with finite elements or finite differences has been carried out in Wegner (2002), Dahlgren (2005), and Wilhelm and Winter (2008). Whereas numerically solving PIDEs are developed by Kjaer (2008), Nguyen and Ehrhardt (2012), and Kudryavtsev and Zanette (2013). Of particular note is the works of Calvo-Garrido et al. (2017) and Calvo-Garrido et al. (2019), where they propose numerical methods based on the semi-Lagrangian sampling method.

In the literature, Fourier-based methods have been employed to treat the swing option valuation problem Jaimungal and Surkov (2011), Zhang and Oosterlee (2013), and Biagini et al. (2015).

Few articles are dedicated to analysing backwards stochastic differential equations; like this, we may find a study of BSDEs with jumps related to swing options in Bernhart et al. (2012). Another work is Rodríguez (2011), which presents a BSDE model to value monthly, daily and hourly. Included the representations of how can use forward contracts and European calls to hedge spot electricity price risks.

A simple fixed price annual swing contract can be viewed as a special case of a gas storage contract in our study we we analysed: Boogert and De Jong (2008), Bardou et al. (2009), Carmona and Ludkovski (2010), Boogert and De Jong (2011), Warin (2012). The curious reader can refer to Warin (2012) devoted to the efficient computation of the deltas to simulate hedging strategies. This article is one of the few that regarding the computation of the hedge on swing options.

Make-up and carry forward describe only a few works, as these aspects further complicate the situation and introduce additional complications. Løland and Lindqvist (2008) describe the make-up right and argue that the buyer has no benefit in speculation on make-up.

Instead, Holden et al. (2011) present an algorithm to evaluate a swing contract with the carry-forward clause and conclude that this right is a substantial increase in the value of the contract. They considered the contract with a fixed number of swing rights and fixed number of carry forward rights. Although the authors only estimate the cost of the contract and do not extract optimal solutions. The paper by Edoli et al. (2013) introduces a new mathematical pricing model for a make-up clause and some instances of a carry-forward clause that can be inserted in a gas swing contract. In their work, the authors treated a 3-years swing contract in discrete time.

The authors' considered the rights separately while Chiarella et al. (2016) evaluate Gas Sales Agreements (GSAs) with both make-up and carry-forward banks in a regime-switching forward price curve model. Dong and Kang (2019) proposes a two-dimensional trinomial tree framework for pricing multiple-year GSAs with make-up, carry forward, and indexation, given knowledge of forward price dynamics of both gas and index. Given the complication of the problem in the presence of make-up and carry forward clauses, it can be concluded that the quantitative literature is limited.

3.4 Conclusion

In conclusion, we note that the use of energy derivatives, such as swing contracts, is primarily due to specific features of the electric power market, such as storage problems of the electricity, volume risks, and high market volatility. These instruments meet complex production, speculative and hedging objectives and requirements of the parties, which allows efficient planning and implementation of transactions in the electricity market.

This study is a review of the academic literature on the valuation of swing options. The studies reviewed to demonstrate the relevance of the swing option valuation. Overall, there is a strong positive trend in the number of articles published over the years. Simultaneously, the growing instability in the consumption of energy resources worldwide makes us expect even more research attention to this topic in the future.

This review contributes to the current literature by providing a complete picture of research into swing options and other variations.

Conclusion

This dissertation focuses on real options and their application. This research aims to analyse energy investments, as well as, to enhance existing real options methods to better decision making. We managed to solve the determined research tasks and questions. Below, we will describe the contribution and results of this research and discussed limitations and future research directions.

Firstly, this research contributes to the renewable support and carbon pricing policies studies by introducing and analysing the support mechanism from a previously not considered side. To the best of our knowledge, the "dirty" option procedure has been presented here for the first time. This research is also the first to numerically analyse the effects of the different support mechanisms on investment decisions with the possibility of investing in traditional energy.

Secondly, the model is presented in the design of a usable algorithm, and its use is illustrated with a numerical example. The method examines both risk-aversion and endogenous capacity decisions at the same time. Although the model has been created without context, its application is not limited to the energy sector but can be extended to investment profitability analysis in general.

Thirdly, the literature review conducted within this research includes more than 100 scientific papers. State-of-the-art swing option research directions are revealed and guide researchers in focusing their future work.

This study creates significant new knowledge for different stakeholders in real options valuation and related areas such as swing options. Any research is limited by the focus taken and by the methods selected - this also applies to this dissertation. One essential issue to recognize when examining this thesis is that the numerical results have been achieved by using a stylised investment case that may or may not apply to the planning of real-world investments. The numerical assumptions made are not necessarily completely reflecting the unfolding future states; however, this limitation is to some extent neutralised by the sensitivity analysis and the uncertainty embedded into modelling.

The models selected are a subset of all real option analysis methods available. Using a more extensive selection of models may bring additional insight that has not come out in this dissertation. The validity of the conclusions made is, to some extent, supported by the already existing articles.

This work can serve as a basis for future research directions. Among the main extensions, we consider using more sophisticated (mathematically speaking) models for the underlying variables. To go one step ahead from Chapter 1, we can consider another price model for valuation that is more suitable for electricity and fuel prices. Thus, the model could be able to fit better the market data. However, it might be more challenging to apply the numerical approach proposed here. From Chapter 2, we can add idiosyncratic risk and risk-aversion in a capacity-setting real options game that is a promising direction for future investigation. We believe that analysing the role of incomplete information in a real options game with endogenous capacity is an important aspect that needs to be studied. From Chapter 3, by analysing the vast amount of swing options literature and understanding the gap in the literature, it is possible to analyse swing options valuation as an impulse control problem, with the ability to create a model that will be more competitive than the classical approach.

Appendix A

Renewable energy investments

A.1 Supplementary data

Symbol	Description	Value	Unit of measure
B	Budget	10000	Thousands of EUR
T	Project's life	20	years
M	Time to maturity of option	30	years
I_D	Marginal cost of installation of dirty energy	900	Thousands of EUR per MW
I_G	Marginal cost of installation of green energy	1600	Thousands of EUR per MW
h_D	Annual capacity factor of D	95	%
$A_D * h_D$	Production coefficient of D	8076 * 0.95	MW*h per MW
h_G	Annual capacity factor of G	34	%
$A_G * h_G$	Production coefficient of G	8076 * 0.34	MW*h per MW
γ	Diseconomies of scale of G	0.9	-

Symbol	Description	Value	Unit of measure
r	Discount rate	0.05	-
μ_E=μ_F	Trend parameter E e F	0	-
μ_G=μ_C	Trend parameter green certificates e carbon permits	0	-
σ_E=σ_F	Volatility parameter E e F	0.06	-
σ_G=σ_C	Volatility parameter green certificates e carbon permits	0.07	-
ρ_{EF}	Correlation coefficient E F	0.7	-
ρ_{EG}=ρ_{FC}	Correlation coefficient	0.6	-
ρ_{FG}=ρ_{EC}	Correlation coefficient	0.5	-
G_{good}	FiT in the good state	0.015	Thousands of EUR per MW*h
G_{bad}	FiT in the bad state	0.01	Thousands of EUR per MW*h
C_{good}	Carbon tax in the good state	0.01	Thousands of EUR per MW*h
C_{bad}	Carbon tax in the bad state	0.015	Thousands of EUR per MW*h
λ_G	Instantaneous transition rate for FiT	0 or 0.1, 0.2	1/years
λ_C	Instantaneous transition rate for Carbon tax	0 or 0.1, 0.2	1/years

A.2 Market and policy uncertainty

Let's fix a two dimensional model[1]:

$$\begin{gathered}\pi_t = Q_G(x_G)(E_t + G_t) + Q_D(x_D)(E_t - F_t - C_t) = \\ E_t(Q_G(x_G) + Q_D(x_D)) - F_t Q_D(x_D) + G_t Q_G(x_G) - C_t Q_D(x_D),\end{gathered} \tag{A.1}$$

where E_t and F_t are GBMs, G_t and C_t are continuous-time Markov chains.

We assume that α_t be time-homogeneous continuous-time Markov with transition intensities λ_α and two states: α_{Good} and α_{Bad}. $\alpha_t = G_t, C_t$

$$\alpha_t = \begin{cases} \alpha_{Bad} & \text{if there was any change of policy in } (0,t] \\ \alpha_{Good} & \text{otherwise} \end{cases}$$

One should note here that α_{Bad} is absorbing state (once reached, the system where leaves).
$P(\alpha_{t+dt} = \alpha_{Bad} | \alpha_t = \alpha_{Good}) = \lambda_\alpha dt + o(dt)$
$P(\alpha_{t+dt} = \alpha_{Good} | \alpha_t = \alpha_{Good}) = 1 - \lambda_\alpha dt + o(dt)$
$P(\alpha_{t+dt} = \alpha_{Bad} | \alpha_t = \alpha_{Bad}) = 1$
$P(\alpha_{t+dt} = \alpha_{Good} | \alpha_t = \alpha_{Bad}) = 0$

We have a generator matrix of the Markov chain $A = \begin{pmatrix} -\lambda & \lambda \\ 0 & 0 \end{pmatrix}$.

The Kolmogorov forward equations can be written as the matrix differential equations $P'(t) = P(t)A$. The system can be solved $P(t) = P(0)e^{tA} = e^{tA}$. We can decompose A into $A = QDQ^{-1}$, where Q consists of the eigenvectors of A and D consists of the eigenvalues of A. In this case, we get $e^{At} = Qe^{Dt}Q^{-1}$, where e^{Dt} is a diagonal matrix. The transition matrix is $P(t) = \begin{pmatrix} e^{-\lambda t} & 1 - e^{-\lambda t} \\ 0 & 1 \end{pmatrix}$.

$$E_{\alpha_{Good}}\left(\int_0^T e^{-rt}\alpha_t dt\right) = \int_0^T e^{-rt} E_\alpha(\alpha_t) dt, \tag{A.2}$$

where $E_{\alpha_{Good}}(\alpha_t) = (1,0) \begin{pmatrix} e^{-\lambda t} & 1 - e^{-\lambda t} \\ 0 & 1 \end{pmatrix} \begin{pmatrix} \alpha_{Good} \\ \alpha_{Bad} \end{pmatrix} =$

[1] Do not consider green certificates and carbon permits

$$= (e^{-\lambda t}, 1 - e^{-\lambda t}) \begin{pmatrix} \alpha_{Good} \\ \alpha_{Bad} \end{pmatrix} = \alpha_{Good} e^{-\lambda t} + \alpha_{Bad}(1 - e^{-\lambda t}).$$

So we have

$$\alpha_{Good} \frac{1 - e^{-(r+\lambda)T}}{r + \lambda} + \alpha_{Bad}\left[\frac{1 - e^{-rT}}{r} - \frac{1 - e^{-(r+\lambda)T}}{r + \lambda}\right] \tag{A.3}$$

We get general pay-off equation for 2D models ($E_0 = E$, $F_0 = F$):

$$\begin{aligned} E(Q_G(x_G) + Q_D(x_D)) \frac{1 - e^{-(r+\mu_E)T}}{r - \mu_E} - F Q_D(x_D) \frac{1 - e^{-(r+\mu_F)T}}{r - \mu_F} + \\ Q_G(x_G)\left[G_{Good} \frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G} + G_{Bad}\left[\frac{1 - e^{-rT}}{r} - \frac{1 - e^{-(r+\lambda_G)T}}{r + \lambda_G}\right]\right] + \\ Q_D(x_D)\left[C_{Good} \frac{1 - e^{-(r+\lambda_C)T}}{r + \lambda_C} + C_{Bad}\left[\frac{1 - e^{-rT}}{r} - \frac{1 - e^{-(r+\lambda_C)T}}{r + \lambda_C}\right]\right] \end{aligned} \tag{A.4}$$

To summarize, replace on $\tilde{r_h}(\mu_h, \lambda) = \frac{1 - e^{-(r+\lambda-\mu_h)T}}{r+\lambda-\mu_h}$,
$\overline{G} = G_{Good}\overline{r}(0, \lambda_G) + G_{Bad}(\overline{r}(0,0) - \overline{r}(0, \lambda_G))$, and
$\overline{C} = C_{Good}\overline{r}(0, \lambda_C) + C_{Bad}(\overline{r}(0,0) - \overline{r}(0, \lambda_C))$.
In the end, we have general pay-off with policy uncertainty:

$$Q_G(x_G)[E\overline{r}(\mu_E, 0) + \overline{G}] + Q_D(x_D)[E\overline{r}(\mu_E, 0) - F\overline{r}(\mu_F, 0) + \overline{C}] \tag{A.5}$$

Appendix B

Proofs

B.1 Proof of Proposition 1

We first rewrite (2.4) as follows:

$$\begin{aligned} F(x, Q_F, Q_L) &= B_F U(Q_F x(1 - \eta(Q_F + Q_L)Q_F)) - U(cQ_F) \\ &= \frac{B_F Q_F^{1-\gamma_F} x^{1-\gamma_F} (1 - \eta(Q_F + Q_L))^{1-\gamma_F}}{1 - \gamma_F} - \frac{c^{1-\gamma_F} Q_F^{1-\gamma_F}}{1 - \gamma_F} \end{aligned} \tag{B.1}$$

where the first equality can be derived by either guessing and verifying that the function $B_F U(\cdot)$ solves the differential equation satisfied by the conditional expected value or, more elegantly, by using theorem 9.18 in Karatsas and Shreve, Methods of Mathematical Finance. First order condition for optimal capacity gives (2.7):

$$B_F x^{1-\gamma_F} (1 - \eta(2Q_F + Q_L))(1 - \eta(Q_F + Q_L))^{-\gamma_F} = c^{1-\gamma_F}. \tag{B.2}$$

Let Q_F be the solution of (B.2) and define X_F as the entry threshold, that is:

$$X_F = \arg\max_{x \geq 0} \frac{F(x, Q_F)}{x^\beta} \tag{B.3}$$

Solving the maximization problem (B.3) gives X_F as a function of Q_F:

$$X_F(Q_F) = \frac{c}{1-\eta(Q_F+Q_L)}\Big(\frac{B_F(\beta+\gamma_F-1)}{\beta}\Big)^{\frac{1}{\gamma_F-1}} \tag{B.4}$$

Now, we insert (B.4) into (B.2) and simplify, to obtain:

$$\gamma_F + \eta(\beta Q_F + Q_F + Q_L) - \gamma_F \eta(Q_F + Q_L) - 1 = 0,$$

from which the (2.9) follows. Inserting (2.9) into (B.4) gives (2.8). Finally, the function $A_F(Q_L)$ is obtained by imposing continuity of the follower's value function at the entry threshold (2.8).

Consider the follower's strategy under three cases. It is worth noting that in the first and second cases, the follower's strategy values do not change. While in the third case, when the leader is a risk-averse, there are changes, since $\gamma_F = 0$. The corresponding equations for principal parameters are given in the table below:

Parameters	$\gamma_L = \gamma_F = \gamma > 0$	$\gamma_F > 0, \gamma_L = 0$	$\gamma_F = 0, \gamma_L > 0$
$\widehat{X}_F(Q_L)$	$\frac{c(1+\beta-\gamma)}{1-\eta Q_L}\big(\frac{B(\beta+\gamma-1)}{\beta^\gamma}\big)^{\frac{1}{\gamma-1}}$	$\frac{c(1+\beta-\gamma_F)}{1-\eta Q_L}\big(\frac{B_F(\beta+\gamma_F-1)}{\beta^{\gamma_F}}\big)^{\frac{1}{\gamma_F-1}}$	$\frac{c(1+\beta)}{(1-\eta Q_L)B_F(\beta-1)}$
$\widehat{Q}_F(Q_L)$	$\frac{(1-\gamma)(1-\eta Q_L)}{\eta(\beta-\gamma+1)}$	$\frac{(1-\gamma_F)(1-\eta Q_L)}{\eta(\beta-\gamma_F+1)}$	$\frac{(1-\eta Q_L)}{\eta(\beta+1)}$

Table B.1: Summary of $\widehat{X}_F(Q_L)$ and $\widehat{Q}_F(Q_L)$ under three cases.

B.2 Proof of Proposition 2

Using theorem 9.18 of Karatsas Shreve again, we rewrite (2.12) as follows:

$$
\begin{aligned}
L^D(x, Q_L) = & B_L U(x(1-\eta Q_L)Q_L)) - U(cQ_L) + (\frac{x}{\widehat{X}_F})^\beta \\
& B_L(U(x(1-\eta(Q_L+\widehat{Q}_F))Q_L) - U(x(1-\eta Q_L)Q_L)) \\
& = \frac{B_L Q_L^{1-\gamma_L} x^{1-\gamma_L}(1-\eta Q_L)^{1-\gamma_L}}{1-\gamma_L} - \frac{c^{1-\gamma_L} Q_L^{1-\gamma_L}}{1-\gamma_L} + \left(\frac{x}{\widehat{X}_F}\right)^\beta \\
& \frac{B_L(Q_L^{1-\gamma_L}\widehat{X}_F^{1-\gamma_L}(1-\eta(\widehat{Q}_F+Q_L))^{1-\gamma_L} - Q_L^{1-\gamma_L}\widehat{X}_F^{1-\gamma_L}(1-\eta Q_L)^{1-\gamma_L})}{1-\gamma_L},
\end{aligned}
\tag{B.5}
$$

where $\widehat{X}_F$ and $\widehat{Q}_F$ are defined in (2.8) and (2.9), respectively. First order condition for maximization with respect to Q_L gives (B.7).

Now define X_L^D as the utility-maximizing threshold, that is the value that solves

$$
\max_{x \geq 0} \frac{L^D(x, Q_L(x))}{x^\beta}, \tag{B.6}
$$

where $Q_L(x)$ is implicitly defined by (2.14). First order condition for maximization in x reads:

$$
\begin{aligned}
& \frac{B_L \widehat{X}_F^{1-\beta-\gamma_L}(\frac{\beta-\beta\eta Q_L(x)}{\beta-\gamma_F+1})^{-\gamma_L}}{\beta-\gamma_F+1}(\gamma_L+\eta(\beta-\gamma_L+1)Q_L(x)-1)\cdot \\
& Q_L'(x)((\beta-\gamma_F+1)(\frac{\beta-\beta\eta Q_L(x)}{\beta-\gamma_F+1})^{\gamma_L} - \beta(1-\eta Q_L(x))^{\gamma_L}) = \\
& c^{-\gamma_L} x^{-\beta-\gamma_L-1}(-cx^{\gamma_L}(1-\eta Q_L(x))^{\gamma_L}((\gamma_L-1)xQ_L'(x)+\beta Q_L(x)) - \\
& B_L x c^{\gamma_L}((\gamma_L-1)x(2\eta Q_L(x)-1)Q_L'(x)+(\beta+\gamma_L-1)Q_L(x)(\eta Q_L(x)-1)))
\end{aligned}
\tag{B.7}
$$

We now insert (2.14) into (B.7) and rearrange terms to obtain

$$
x = \left(\frac{B_L(\beta+\gamma-1)c^{\gamma_L-1}(1-\eta Q_L(x))^{1-\gamma_L}}{\beta}\right)^{\frac{1}{\gamma_L-1}}. \tag{B.8}
$$

To obtain the desired $\widehat{Q}_L^D, \widehat{X}_L^D$, insert (B.8) into (2.14), rearrange terms and solve for Q_L. Then insert the solution obtained into (B.8).

Finally, we determine the interval in which the entry deterrence strategy can be applied. x_1^D is the lowest value of the random shock for which the leader invests in positive capacity. To derive x_1^D, it is we insert $Q_L = 0$ into (2.14) and obtain that x_1^D solves the following implicit equation:

$$\frac{c^{1-\gamma_L}}{B_L} = (x_1^D)^{1-\gamma_L} + (x_1^D)^{\beta} \widehat{X}_F(0)^{1-\beta-\gamma_L} \left(\left(\frac{\beta}{\beta - \gamma_F + 1} \right)^{1-\gamma_L} - 1 \right). \tag{B.9}$$

x_2^D is the highest value which makes the follower invest later than the leader. It can be derived by inserting (2.13) into (2.14).

In the symmetric case, we obtain:

$$\widehat{X}_L^D = \frac{c(\beta - \gamma + 1) \left(\frac{B(\beta+\gamma-1)}{\beta} \right)^{\frac{1}{\gamma-1}}}{\beta}, \widehat{Q}_L^D = \frac{1-\gamma}{\eta(\beta - \gamma + 1)}.$$

x_1^D is defined as follows:

$$\frac{c^{1-\gamma}}{B_L} = (x_1^D)^{1-\gamma} + \left(\frac{\beta}{\beta - \gamma + 1} \right)^{1-\gamma} \frac{\left((-\beta + \gamma - 1)(\frac{\beta}{\beta-\gamma+1})^{\gamma} + \beta \right) (x_1^D)^{\beta} \widehat{X}_F(0)^{1-\beta-\gamma}}{\beta}. \tag{B.10}$$

Cumbersome but straightforward algebra leads to:

$$x_2^D = \frac{cr(\beta - \gamma + 1)^2 \left(\beta^{\gamma}(\beta - \gamma + 1)^{\gamma}(\beta + \gamma - 1) - \beta \right) \left(B\rho\beta^{-\gamma}(\beta + \gamma - 1) \right)^{\frac{1}{\gamma-1}}}{\beta^{\gamma}(\beta + \gamma - 1)(\beta - \gamma + 1)^{\gamma-1} - \beta^2 + (\gamma - 1)(\beta + \gamma - 1)(\beta - \gamma + 1)^{2\gamma}}. \tag{B.11}$$

Equation (B.11) gives x_2^D in the symmetric case. Consider changes in the values of the leader's threshold and quantity, as well as x_1^D and x_2^D for the asymmetric case.

In the asymmetric case, when the follower is risk-averse ($\gamma_F > 0$), and the leader is risk-neutral ($\gamma_L = 0$). We rewrite (B.5) as follows:

$$L^D(x, Q_L) = B_L Q_L x(1 - \eta Q_L) - cQ_L + \left(\frac{x}{\widehat{X}_F}\right)^\beta \\ B_L(Q_L \widehat{X}_F(1 - \eta(\widehat{Q}_F + Q_L)) - Q_L \widehat{X}_F(1 - \eta Q_L)), \tag{B.12}$$

where $\widehat{X}_F$ and $\widehat{Q}_F$ are defined in (2.8) and (2.9), respectively.

Repeating the process of deriving the formulas described above, we obtain the following values of $\widehat{X}_L^D$ and $\widehat{Q}_L^D$:

$$\widehat{X}_L^D = \frac{c(\beta + 1)}{B_L(\beta - 1)}, \widehat{Q}_L^D = \frac{1}{\eta(\beta + 1)}. \tag{B.13}$$

Finally, we determine the interval in which the entry deterrence strategy can be applied for this case:

$$\frac{c}{B_L} = x_1^D + (x_1^D)^\beta \widehat{X}_F(0)^{1-\beta} \left(\frac{\gamma_F - 1}{\beta - \gamma_F + 1}\right). \tag{B.14}$$

$$x_2^D = \\ \frac{c\,(\beta - \gamma_F + 1)\left(B_L\,(\beta + (\beta - 1)\gamma_F + 1)\left(\frac{B_F(\beta+\gamma_F-1)}{\beta\gamma_F}\right)^{\frac{1}{\gamma_F - 1}} - 1\right)}{B_L\,((\beta - 1)\gamma_F + 1)}. \tag{B.15}$$

In the asymmetric case, when the leader is risk-averse ($\gamma_L > 0$), and the follower is risk-neutral ($\gamma_F = 0$), we have equation (B.5) without change, but $\widehat{X}_F$ and $\widehat{Q}_F$ are defined in TableB.1.

We obtain the same values of $\widehat{X}_L^D$ and $\widehat{Q}_L^D$ are defined in (2.15) and (2.16), respectively. After that, we determine the interval in which the entry deterrence strategy can be applied. x_1^D is defined as follows:

$$\frac{c^{1-\gamma_L}}{B_L} = (x_1^D)^{1-\gamma_L} + (x_1^D)^\beta \widehat{X}_F(0)^{1-\beta-\gamma_L} \left(\left(\frac{\beta}{\beta + 1}\right)^{1-\gamma_L} - 1\right). \tag{B.16}$$

And x_2^D is defined as follows:

$$x_2^D = \frac{c\,(1-\gamma_L)\,\beta^{\gamma_L}(\beta+1)^{\gamma_L+1}\,((\beta-1)B_F)^{\,1-\gamma_L}}{B_F B_L\,((1-\beta)\beta^2(\beta+1)^{\gamma_L}+(\beta^2-1)\,(\beta+\gamma_L-1)\,\beta^{\gamma_L})} + \frac{(\beta+1)c\,((\beta+1)\beta^{\gamma_L}\,(\beta+\gamma_L-1)-\beta(\beta+1)^{\gamma_L}\,(\beta-\gamma_L+1))}{B_F\,((1-\beta)\beta^2(\beta+1)^{\gamma_L}+(\beta^2-1)\,(\beta+\gamma_L-1)\,\beta^{\gamma_L})}. \tag{B.17}$$

B.3 Proof of Proposition 3

The function to be optimized over Q_L in (2.18) reads:

$$L^A(x, Q_L) = \frac{B_L Q_L^{1-\gamma_L} x^{1-\gamma_L}(1-\eta(Q_F(Q_L)+Q_L))^{1-\gamma_L}}{1-\gamma_L} - \frac{c^{1-\gamma_L} Q_L^{1-\gamma_L}}{1-\gamma_L}. \tag{B.18}$$

where we stress that Q_F depends on Q_L. First order conditions for maximization over Q_L reads:

$$\begin{aligned} c^{1-\gamma_L} Q_L^{-\gamma_L} &= B_L Q_L^{-\gamma_L} x^{1-\gamma_L}(1-\eta(Q_F(Q_L)+Q_L))^{1-\gamma_L} \\ &- B_L \eta Q_L^{1-\gamma_L} x^{1-\gamma_L}(Q'_F(Q_L)+1)(1-\eta(Q_F(Q_L)+Q_L))^{-\gamma_L}. \end{aligned} \tag{B.19}$$

Equation (B.2) defines the optimal response of the follower Q_F when the leader chooses capacity Q_L. For each level of the random shock x, the optimal capacities solve (B.2) and (B.19) simultaneously.

By using the implicit function theorem, we can compute $Q'_F(Q_L)$ in its implicit form:

$$Q'_F(Q_L) = \Phi(Q_L) = \frac{\gamma_F + \gamma_F \eta(-(2Q_F(Q_L)+Q_L)) + \eta(Q_F(Q_L)+Q_L) - 1}{\gamma_F(2\eta Q_F(Q_L) + \eta Q_L - 1) - 2\eta(Q_F(Q_L)+Q_L) + 2} \tag{B.20}$$

By inserting (B.20) into (B.19) and rearranging terms, we obtain (2.20).

In the symmetric case, set $\gamma_L = \gamma_F = \gamma$. First order conditions for maximization over Q_L reads:

$$\begin{aligned} c^{1-\gamma} Q_L^{-\gamma} &= B Q_L^{-\gamma} x^{1-\gamma}(1-\eta(Q_F(Q_L)+Q_L))^{1-\gamma} \\ &- B \eta Q_L^{1-\gamma_L} x^{1-\gamma}(Q'_F(Q_L)+1)(1-\eta(Q_F(Q_L)+Q_L))^{-\gamma}. \end{aligned} \tag{B.21}$$

The implicit form of $Q'_F(Q_L)$ is:

$$Q'_F(Q_L) = \Phi(Q_L) = \frac{\gamma + \gamma\eta(-(2Q_F(Q_L)+Q_L)) + \eta(Q_F(Q_L)+Q_L) - 1}{\gamma(2\eta Q_F(Q_L) + \eta Q_L - 1) - 2\eta(Q_F(Q_L)+Q_L) + 2} \tag{B.22}$$

Now, insert (B.22) into (B.21) and put the resulting expression in (B.2).

After rearranging terms, we get:

$$2(\gamma-1)\eta Q_F(Q_L)^2 + Q_F(Q_L)(-\gamma+(\gamma-1)\eta Q_L+2) + Q_L(\eta Q_L-1) = 0, \tag{B.23}$$

which is a quadratic equation in the unknown Q_F. Among the two roots of this equation, the smallest is not admissible, since $(1-\eta(Q_F(Q_L)+Q_L))^{-\gamma}$ is not defined. The largest root gives the explicit expression:

$$Q_F(Q_L) = \Psi(Q_L) = \frac{\gamma-\gamma\eta Q_L+\eta Q_L-2}{4(\gamma-1)\eta} + \frac{\sqrt{(-\gamma+(\gamma-1)\eta Q_L+2)^2-8(\gamma-1)\eta Q_L(\eta Q_L-1)}}{4(\gamma-1)\eta} \tag{B.24}$$

gives the function $\Psi(\cdot)$ in (2.21). To obtain the implicit equation defined by (2.7) and (2.20), use the function $\Phi(\cdot)$ defined (B.20) into (B.19).

In the asymmetric case, when the follower is risk-averse ($\gamma_F > 0$), and the leader is risk-neutral ($\gamma_L = 0$). First order conditions for maximization over Q_L reads:

$$\frac{c}{B_L x} = (1-\eta(Q_F(Q_L)+Q_L)) - \eta Q_L(Q_F'(Q_L)+1). \tag{B.25}$$

We insert (B.20) into (B.25) and, among the two roots, the smallest is not admissible since $(1-\eta(Q_F(Q_L)+Q_L))$ and $Q_F(Q_L)$ are not positive. The largest root gives the explicit expression:

$$Q_F(Q_L) = \Psi(Q_L) = \frac{\eta x B_L\left(3\gamma_F(1-\eta Q_L)+5\eta Q_L-4\right)-2c\eta(\gamma_F-1)}{4\eta^2 x B_L(\gamma_F-1)} + \frac{\left(4cxB_L(\gamma_F-1)\left(\gamma_F(\eta Q_L-1)-\eta Q_L\right)+x^2B_L^2\left(\gamma_F(\eta Q_L-1)+\eta Q_L\right)^2+4c^2(\gamma_F-1)^2\right)^{\frac{1}{2}}}{4\eta x B_L(\gamma_F-1)}, \tag{B.26}$$

where

$$
\begin{aligned}
&0 < Q_L < \frac{\gamma_F - 2}{\eta\,(\gamma_F - 3)},\\
&x > \frac{c\,(2-\gamma_F)}{B_L\,(\gamma_F(\eta Q_L - 1) - 3\eta Q_L + 2)}.
\end{aligned}
$$

In the asymmetric case, when the leader is risk-averse ($\gamma_L > 0$), and the follower is risk-neutral ($\gamma_F = 0$). In this case, (B.20) is equal $Q'_F(Q_L) = -\frac{1}{2}$. By inserting $Q'_F(Q_L) = -\frac{1}{2}$ into (B.19) and rearranging terms we obtain:

$$
x^{\gamma_L - 1} = -\frac{1}{2}B_L c^{\gamma_L - 1}(\eta(2Q_F(Q_L) + 3Q_L) - 2)(1 - \eta(Q_F(Q_L) + Q_L))^{-\gamma_L}. \tag{B.27}
$$

Now, we insert (B.27) into (B.2), we obtain:

$$
(1 - \eta(Q_F(Q_L) + Q_L))^{-\gamma_L} = \frac{2B_F^{1-\gamma_L}(\eta(2Q_F(Q_L) + Q_L) - 1)(1 - \eta(2Q_F(Q_L) + Q_L))^{-\gamma_L}}{B_L(\eta(2Q_F(Q_L) + 3Q_L) - 2)} \tag{B.28}
$$

By inserting (B.28) into (B.27) and rearranging terms we obtain:

$$
Q_F(Q_L) = \frac{1 - \eta Q_L - \frac{c}{xB_F}}{2\eta}. \tag{B.29}
$$

where

$$
\begin{aligned}
&0 < Q_L < \frac{1 - \frac{c}{xB_F}}{\eta},\\
&x > \frac{c}{B_F}.
\end{aligned}
$$

Appendix C

Swing option: Tabulated summary of reviewed papers

№	Authors	Year	Price model	Formulation	Numerical method	Reference
1	Ahmadi, Z., Hosseini, S.M., Bastani, A.F.	2021	MRJ	DP	MCR, TR	(2)
2	Bernal, F., Gobet, E., Printems, J.	2020	MR	PDE	MCR	(26)
3	De Donno, M., Palmowski, Z., Tumilewicz, J.	2020	BS, Lévy	DP	Optimal stopping	(57)
4	Dong, W., Kang, B.	2020	MR	DP	MCR, TR	(62)
5	Calvo-Garrido, M.C., Ehrhardt, M., Vázquez, C.	2019	MRJ	PIDE	Semi-Lagrangian method	(39)
6	Lars Kirkby, J., Deng, S.-J.	2019	MR, Lévy	DP	Optimal stopping, Fourier techniques	(105)
7	Dong, W., Kang, B.	2019	MR	DP	TR	(61)

8	Kohrs, H., Mühlichen, H., Auer, B.R., Schuhmacher, F.	2019	GBM	DP	MCR	(96)
9	Shao, L., Xiang, K.	2019	MR, RS	DP	TR	(136)
10	Berger, B., Dietrich, M., Döttling, R., Heider, P., Spanderen, K.	2018	GBM	Dual method	MCR	(25)
11	De Angelis, T., Kitapbayev, Y.	2018	BS, GBM		Optimal stopping	(56)
12	Fanelli, V., Ryden, A.K.	2018	MR		MCR	(66)
13	Kao, E. P. C., Wang, M.	2018	BS	DP	FE	(89)
14	Shao, L., Xiang, K., Song, Y.	2018	MR, RS	VI		(137)
15	Bender, C., Dokuchaev, N.	2017		BSPDE	Optimal stopping	(21)
16	Calvo-Garrido, M.C., Ehrhardt, M., Vázquez, C.	2017	MR	PDE	Semi-Lagrangian method	(38)
17	Bender, C., Dokuchaev, N.	2016	BS	BSPDE	Stochastic optimal control	(20)
18	Chiarella, C., Clewlow, L., Kang, B.	2016	MR, RS	DP	TR	(44)
19	Klimešová, A., Václavík, T.	2016			MCR	(95)
20	Latifa, I.B., Bonnans, J.F., Mnif, M.	2016	Lévy	DP, VI	Q	(107)
21	Li, L., Qu, X., Zhang, G.	2016		DP	Eigenfunction expansion	(112)
22	Naouara, N. J. B., Trabelsi, F.	2015		DP	Optimal stopping	(124)

23	Kulikov, A., Malykh, N.	2015	Cox-Ross-Rubinstein		TR	(102)
24	Naouara, N. J. B., Trabelsi, F.	2015	GBM	DP	Optimal stopping	(125)
25	Latifa, I.B., Bonnans, J.F., Mnif, M.	2015	MRJ	VI	Optimal stopping	(106)
26	Biagini, F., Bregman, J., Meyer-Brandis, T.	2015	Lévy		Fourier techniques	(28)
27	Bender, C., Schoenmakers, J., Zhang, J.	2015		Dual method	MCR	(22)
28	Müller, J., Hirsch, G., Müller, A.	2015			MCR	(122)
29	Li, H., Ware, A., Guo, L., Chen, W.N.	2014	MR	VI	Wavelet collocation method	(111)
30	Eriksson, M., Lempa, J., Nilssen, T.K.	2014	MD [1] Lévy	DP, PDE	FD	(65)
31	Basei, M., Cesaroni, A., Vargiolu, T.	2014		VI	AT [2]	(15)
32	Kitapbayev, Y., Moriarty, J., Mancarella, P., Blochle, M.	2013	MR	DP	MCR	(92)
33	Aydın, N.S., Rainer, M.	2013	MR		TR	(7)
34	Zhang, B., Oosterlee, C.W.	2013	MR, Lévy	DP	Fourier techniques	(159)
35	Kudryavtsev, O., Zanette, A.	2013	Lévy	DP, PIDE	FD, Wiener–Hopf factorization	(101)

[1] Multidimensional
[2] Approximate techniques

36	Edoli, E., Fiorenzani, S., Ravelli, S., Vargiolu, T.	2013	MR	DP	TR	(64)
37	Turboult, F., Youlal, Y.	2012		DP	MCR	(149)
38	Bernhart, M., Pham, H., Tankov, P., Warin, X.	2012	BS	BSDE	MCR	(27)
39	Wiebauer, K.	2012	GBM		MCR	(155)
40	Bian, Q., Lu, Z.	2012	MR	DP	TR	(29)
41	Nguyen, M.H., Ehrhardt, M.	2012	GBM	PIDE	FD	(127)
42	Benth, F. E., Lempa, J., Nilssen, T. K.	2012	MR	PDE	FD	(24)
43	Warin, X.	2012	2-MR	PDE	MCR	(153)
44	Jaimungal, S., Surkov, V.	2011	MRJ, Lévy	PIDE	MCR, Fourier techniques	(88)
45	Bender, C.	2011	MR	Dual method	MCR	(19)
46	Marshall, T.J., Reesor, R.M., Cox, M.	2011	GBM	DP	TR	(118)
47	Wahab, M.I.M., Lee, C.-G.	2011	RS (GBM)	DP	TR	(151)
48	Rodríguez, J.F.	2011	2-GBM	BSDE		(134)
49	Holden, L., Løland, A., Lindqvist, O.	2011	MR		MCR	(78)
50	Bender, C.	2011	MR	Dual method	MCR	(18)
51	Tashiro, Y.	2011	MR	MP [3]	TR	(141)
52	Marshall, T.J., Reesor, R.M.	2011	5-GBM	DP	TR	(117)

[3] Mathematical programming

53	Boogert, A., De Jong, C.	2011	3-GBM	DP	MCR	(32)
54	Pages, G., Wilbertz, B.	2010	2-MR	DP	Q, TR	(129)
55	Bronstein, A.L., Pagés, G., Wilbertz, B.	2010		DP	Q	(37)
56	Wahab, M.I.M., Yin, Z., Edirisinghe, N.C.P.	2010	RS	DP	TR	(152)
57	Bardou, O., Bouthemy, S., Pagès, G.	2010	2-MR	DP	Q, TR	(13)
58	Aleksandrov, N., Hambly, B.M.	2010	MR	DP, Dual method	MCR	(6)
59	Becker, M.	2010	Variance gamma process	Difference-of-gammas bridge sampling	MCR	(17)
60	Kiesel, R., Gernhard, J., Stoll, S.-O.	2010	MR	VI	FD, MCR	(91)
61	Carmona, R., Ludkovski, M.	2010	RS	DP, VI	MCR	(41)
62	Bardou, O., Bouthemy, S., Pagès, G.	2009	2-MR	DP	Q	(12)
63	Haarbrücker, G., Kuhn, D.	2009	2-MRJ (Forward price model)	Stochastic DP	TR	(72)
64	Hambly, B., Howison, S., Kluge, T.	2009	2-MRJ	DP	TR	(74)
65	Hirsch, G.	2009	RS		MCR	(77)

66	Steinbach, M. C., Vollbrecht, H.-J.	2009		Stochastic programming	TR (fan)	(140)
67	Breslin, J., Clewlow, L., Strickland, C., van der Zee, D.	2008				(35)
68	Breslin, J., Clewlow, L., Strickland, C., van der Zee, D.	2008				(36)
69	Kjaer, M.	2008	MRJ	DP, PIDE	FD, Fourier techniques	(94)
70	Geman, H., Kourouvakalis, S.	2008	MRJ	DP	TR, Q	(70)
71	Carmona, R., Dayanik, S.	2008	MR, GBM	DP	Optimal stopping	(40)
72	Carmona, R., Touzi, N.	2008	BS, GBM	DP, VI	Mal, Optimal stopping	(42)
73	Bjerksund, P., Myksvoll, B., Stensland, G.	2008		DP		(30)
74	Wilhelm, M., Winter, C.	2008	BS	PDE	FE	(156)
75	Boogert, A., De Jong, C.	2008	MR(J)	PDE	MCR	(31)
76	Ross, S. M., Zhu, Z.	2008	GBM	DP	AT	(135)
77	Zeghal, A.B., Mnif, M.	2006	Lévy	DP	Mal	(158)
78	Hinz, J.	2006			TR	(76)

79	Barrera-Esteve, C., Bergeret, F., Dossal, C., (...), Munos, R., Reboul-Salze, D.	2006	MR	DP	MCR (Parametric approximation)	(14)
80	Baldick, R., Kolos, S., Tompaidis, S.	2006	Demand / supply model	Stochastic DP		(9)
81	Figueroa, M.	2006	MRJ		MCR	(67)
82	Thanawalla, R. K.	2006	MR, GBM		MCR	(142)
83	Dahlgren, M.	2005		quasi-VI, PDE		(51)
84	Dahlgren, M., Korn, R.	2005	BS	VI	FD	(52)
85	Meinshausen, N., Hambly, B.M.	2004	MR		MCR	(121)
86	Jaillet, P., Ronn, E.I. and Tompaidis, S.	2004	1-MR (with seasonality)	DP	TR	(87)
87	Ibáñez, A.	2004	Lognormal and MR	DP	MCR (with optimal exercise frontier)	(84)
88	Keppo, J.	2004		DP	AT	(90)
89	Davison, M., Anderson, L.	2003	Mixture of Poissons	DP	MCR	(55)
90	Dörr, U.	2003	MR	DP	MCR	(63)
91	Clewlow, L., Strickland, C. and Kaminski, V.	2001			TR	(50)
92	Lari-Lavassani, A., Simchi, M., Ware, A.	2001	(1- and 2-) MR	DP	TR	(104)

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