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The next-to-leading order jet vertex for Mueller-Navelet jets

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Introduction

An important manifestation of the BFKL [1] dynamics at hadron colliders such as TEVATRON and LHC is the so called Mueller-Navelet process [2]. It is an inclusive production at high energies of two forward high-k_T jets in proton-proton collision:

$$p(p_1) + p(p_2) \to \text{jet}(k_1) + \text{jet}(k_2) + X \ , \tag{1}$$

which are detected in the fragmentation regions of two colliding protons $p(p_1)$ and $p(p_2)$.

If the rapidity gap between the two produced jets is large in the center of mass (CM) frame, at large energy $\sqrt{s}$ the BFKL resummation comes into play, since large logarithms of the energy compensate the small QCD coupling and must be resummed to all orders in perturbation theory. On the other side, the process is started by two hadrons each emitting one parton, according to its parton distribution function (PDF), and collinear factorization allows to systematically resum the logarithms of the hard scale, calculating the standard DGLAP evolution [3] of the PDFs.

The BFKL approach provides a general framework for the resummation of energy logarithms in the leading logarithmic approximation (LLA), which means resummation of all terms $(\alpha_s \ln(s))^n$, and in the next-to-leading logarithmic approximation (NLA), which means resummation of all terms $\alpha_s(\alpha_s \ln(s))^n$. Indipendently from the process, the interaction of two Reggeized gluons implies a resummation which leads to consider a Green's function determined through the BFKL equation. The BFKL equation is an iterative integral equation, whose kernel is known at the next-to-leading order (NLO) both for forward scattering [4, 5] and for any momentum transfer t (not growing with energy)

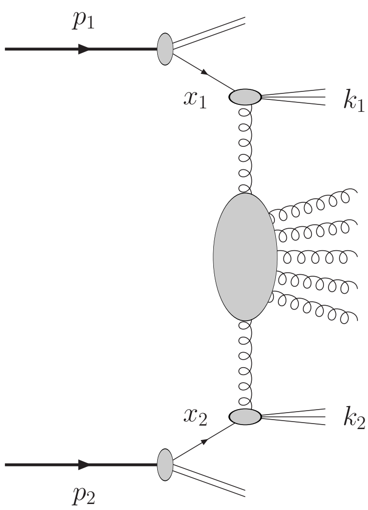

Figure 1: Diagrammatical representation of jets production process

and any possible two-gluons color state in the t-channel [6].
The other components for a complete description of a process in the BFKL approach are the impact factors; among these, those calculated with NLO accuracy are those for colliding quark and gluons [10, 11, 12, 13], for forward jet production [7, 8], for the $\gamma^* \to \gamma^*$ transition [14] and for the γ^* to light vector meson transition at leading twist [15].

On the other hand the full NLO Green's function was implemented [16, 17, 18] and effects related with QCD running coupling were studied in [19, 20]. These improvements brought to a better description of the results of the D0 collaboration at TEVATRON [21]. In fact these results revealed a stronger rise of the Mueller-Navelet jet cross section with energy than predicted by LLA BFKL calculations. A complete analysis of the process (1) was made [22], which incorporates NLO correction to both the BFKL Green's function and the jet impact factors, calculated in [7, 8]. One of the results of this paper is that for kinematics typical of the LHC experiment the effect of NLO corrections to the jet impact factor is very important, of the same order as the one obtained from the NLO corrections to Green's function.

This observation is similar to one obtained earlier in the NLA analysis of the diffractive double ρ-electroproduction [23]. Another important conclusion of [22] is that the results for Mueller-Navelet jet observables obtained within complete NLA BFKL analysis appeared to be very close to the one calculated in the conventional collinear factorization at the NLO, with the only exception of the ratio between the azimuthal angular moments $\langle \cos(2\phi) \rangle / \langle \cos \phi \rangle$. It is important to have an independent calculation of Mueller-Navelet jet observables in NLA. The aim of the present thesis is the calculation of NLO correction to the jet impact factor in order to have an independent check of the results of [7, 8]. This will be developed in the second chapter, where the results obtained in collaboration with F. Caporale, D.Yu. Ivanov, B. Murdaca and A. Papa [24] will be presented. Here there will be the advantage of starting from the general definition for the impact factors at NLO, see [25], which allows to come to the results more shortly than in [7, 8] and in a more general way.

Chapter 1

The BFKL approach

In perturbative QCD for a scattering process in a dynamical regime with large energy $\sqrt{s}$ and fixed squared momentum transfer t (i.e. not growing with s), the BFKL theory prescribes that if the process happens with the exchange of quantum numbers of gluon in t-channels we have an amplitude that goes like $s^{j(t)}$. This effect is called "Reggeization" of the gluon. The function $j(t)$ is the Regge trajectory of the particle. Further, and this is characteristic of QCD, the Reggeon gives the leading contribution in each order of perturbation theory. If we consider the elastic process (see, for instance, Ref. [26]) $A+B \longrightarrow A'+B'$ with exchange of gluon quantum numbers in the t-channel, i.e. for octet color representation in the t-channel and negative signature, gluon Reggeization means that, in the Regge kinematical region $s \simeq -u \to \infty, t$ fixed (i.e. not growing with s), the amplitude of this process takes the form

$$\left(A_8^-\right)_{AB}^{A'B'} = \Gamma_{A'A}^{\mathcal{C}} \left[\left(\frac{-s}{-t} \right)^{j(t)} - \left(\frac{s}{-t} \right)^{j(t)} \right] \Gamma_{B'B}^{\mathcal{C}}. \tag{1.1}$$

Here c is a color index and $\Gamma_{P'P}^{\mathcal{C}}$ are the particle-particle-Reggeon vertices, not depending on s. They can be written as $\Gamma_{P'P}^{\mathcal{C}} = g\langle P'|T^{\mathcal{C}}|P\rangle\Gamma_{P'P}$, where g is the QCD coupling constant and T^c are the color group generators in the fundamental (adjoint) representation for quarks (gluons). This form of the amplitude has been proved rigorously [27] to all orders of perturbation theory in the leading-logarithmic approximation (LLA), which means resummation of the terms $\alpha_s^n \ln^n s$. In this approximation $\Gamma_{P'P}$ is given simply by $\delta_{\lambda_{P'}\lambda_P}$, where λ_P is the helicity of the particle P, and ω gives for the Reggeized gluon trajectory (which enters with 1-loop accuracy) the following result [28],

$$\omega^{(1)}(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N}{2} \int \frac{d^{D-2}k_\perp}{k_\perp^2 (q-k)_\perp^2} = -\frac{g^2 N \Gamma(1-\epsilon)}{(4\pi)^{D/2}} \frac{\Gamma^2(\epsilon)}{\Gamma(2\epsilon)} (-q_\perp^2)^\epsilon .$$

Here $D = 4 + 2\epsilon$ has been introduced in order to regularize the infrared divergences and the integration is performed in the space transverse to the momenta of the initial colliding particles.

1.1 BFKL in the LLA

Through the use of unitarity relation the BFKL approach allows to obtain amplitudes in $t-$channel with quantum numbers different from the gluon ones. In the LLA, the main contributions to the unitarity relations from inelastic amplitudes come from the multi-Regge kinematics, i.e. when rapidities of the produced particles are strongly ordered and their transverse momenta do not grow with s. In the multi-Regge kinematics, the real part of the production amplitudes takes a simple factorized form, due to gluon Reggeization,

$$A_{AB}^{\tilde{A}\tilde{B}+n} = 2s\Gamma_{\tilde{A}A}^{\mathcal{C}1} \left(\prod_{i=1}^{n} \gamma_{c_i c_i+1}^{P_i}(q_i, q_i+1) \left(\frac{s_i}{s_R}\right)^{\omega_i} \frac{1}{t_i} \right) \frac{1}{t_{n+1}} \left(\frac{s_{n+1}}{s_R}\right)^{\omega_{n+1}} \Gamma_{\tilde{B}B}^{\mathcal{C}_{n+1}}, \tag{1.2}$$

where s_R is an energy scale, irrelevant in the LLA, $\gamma_{c_i c_i+1}^{P_i}(q_i, q_i+1)$ is the effective vertex for the production of the particles P_i with momenta $k_i = q_i - q_{i+1}$ in the collisions of Reggeons with momenta q_i and $-q_{i+1}$ and color indices c_i and c_{i+1},

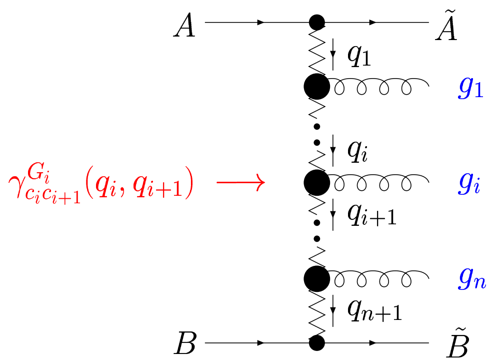

Figure 1.1: Diagrammatical representation of inelastic amplitude in LLA

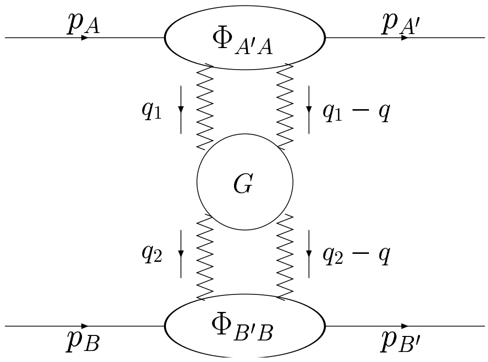

Figure 1.2: Diagrammatical representation of a BFKL process, the ovals are the impact factors of particle A and B the circle is the Green's function of the Reggeon-Reggeon scattering

$q_0 \equiv p_A$, $q_{n+1} \equiv -p_B$, $s_i = (k_{i-1} + k_i)^2$, $k_0 \equiv p_{\tilde{A}}$, $k_{n+1} \equiv p_{\tilde{B}}$ and ω_i stands for $\omega(t_i)$, with $t_i = q_i^2$. A schematic view of this amplitude is in Fig. (1.1). In the LLA, P_i can be only the state of a single gluon. By using s-channel unitarity and the previous expression for the production amplitudes, the amplitude of the elastic scattering process $A + B \longrightarrow A' + B'$ at high energies can be written as

$$\begin{aligned} A_{AB}^{A'B'} &= \frac{is}{(2\pi)^{D-1}} \int \frac{d^{D-2}q_1}{\vec{q}_1^{\,2}\vec{q}_1^{\,\prime 2}} \int \frac{d^{D-2}q_2}{\vec{q}_2^{\,2}\vec{q}_2^{\,\prime 2}} \int_{\delta-i\infty}^{\delta+i\infty} \frac{d\omega}{\sin(\pi\omega)} \sum_{R,\nu} \Phi_{A'A}^{(R,\nu)}(\vec{q}_1; \vec{q}; s_0) \\ &\times \left[\left(\frac{-s}{s_0} \right)^{\omega} - \tau \left(\frac{s}{s_0} \right)^{\omega} \right] G_{\omega}^{(R)}(\vec{q}_1, \vec{q}_2, \vec{q}) \Phi_{B'B}^{(R,\nu)}(-\vec{q}_2; -\vec{q}; s_0). \end{aligned} \tag{1.3}$$

The term $\Phi_{P'P}^{(R,\nu)}$ is the so-called impact factor in the t-channel color state (R, ν) of which a particular case will be the object of this thesis and $G_{\omega}^{(R)}$ is the Mellin transform of the Green's functions for Reggeon-Reggeon scattering. Here $q_1' \equiv q_i - q, q \sim q_\perp$ is the momentum transfer in the process, the sum is over the irreducible representations R of the color group contained in the product of two adjoint representations and over the states ν of these representations, τ is the signature equal $+1(-1)$ for symmetric (antisymmetric) representations and s_0 is an energy scale. The dependence on s is determined by $G_{\omega}^{(R)}$, which obeys

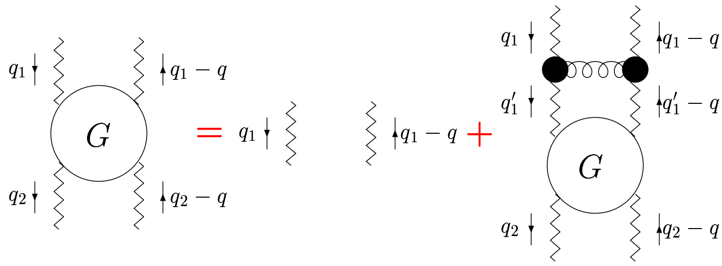

Figure 1.3: BFKL equation in LLA in schematical representation

the equation

$$\begin{aligned}\omega G_\omega^{(R)}(\vec{q}_1, \vec{q}_2, \vec{q}\,) &= \vec{q}_1^{\,2}\vec{q}_1^{\,\prime 2}\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) \\ &+ \int \frac{d^{D-2}q_r}{\vec{q}_r^{\,2}\vec{q}_r^{\,\prime 2}} K^{(R)}(\vec{q}_1, \vec{q}_r; \vec{q}\,) G_\omega^{(R)}(\vec{q}_r, \vec{q}_2; \vec{q}\,),\end{aligned} \tag{1.4}$$

whose integral kernel is

$$K^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,) = [\omega(-\vec{q}_1^{\,2}) + \omega(-\vec{q}_1^{\,\prime 2})]\delta^{(D-2)}(\vec{q}_1 - \vec{q}_2) + K_r^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,), \tag{1.5}$$

is composed by a “virtual” part

$$[\omega(-\vec{q}_1^{\,2}) + \omega(-\vec{q}_1^{\,\prime 2})]$$

related to the gluon trajectory, and by a “real” part

$$K_r^{(R)}(\vec{q}_1, \vec{q}_2; \vec{q}\,)$$

related to particle production in Reggeon-Reggeon collisions. In the LLA, the “virtual” part of the kernel takes contribution from the gluon Regge trajectory with 1-loop accuracy, $\omega^{(1)}$, while the “real” part takes contribution from the production of one gluon in the Reggeon-Reggeon collision at Born level, $K_{RRG}^{(B)}$. The BFKL equation is the one given in Eq. (1.4) and is valid for $t = 0$ and singlet quantum numbers in the t-channel; it can be schematized as in Fig. (1.3).

Figure 1.4: Diagrammatical representations of replacements $\Gamma_{P'P}^{\mathcal{C}(Born)} \to \Gamma_{P'P}^{\mathcal{C}(1-loop)}$(a) and $\gamma_{c_i c_i+1}^{G_i(Born)} \to \gamma_{c_i c_i+1}^{G_i(1-loop)}$(b).

1.2 BFKL in the NLA

In the NLA, the Regge form of the elastic amplitude (1.1) and of the production amplitudes (1.2), implied by gluon Reggeization, has been checked only in the first three orders of perturbation theory [29].

In order to derive the BFKL equation in the NLA, gluon Reggeization is assumed to be valid to all orders of perturbation theory. Recently it has been shown that Reggeization is fulfilled also in the NLA, through the study of the so-called "bootstrap" conditions [30].

In the NLA it is necessary to include into the unitarity relations contributions which differ from those in the LLA by having one additional power of α_s or one power less in $\ln s$. The first set of corrections is realized by performing, only in one place, one of the following replacements in the production amplitudes (1.2) entering the s-channel unitarity relation:

$$\omega^{(1)} \longrightarrow \omega^{(2)}, \qquad \Gamma_{P'P}^{\mathcal{C}(Born)} \longrightarrow \Gamma_{P'P}^{\mathcal{C}(1-loop)}, \qquad \gamma_{c_i c_i+1}^{G_i(Born)} \longrightarrow \gamma_{c_i c_i+1}^{G_i(1-loop)}$$

the last two replacements are diagrammatically shown in Figs. (1.4(a), 1.4(b)).

The second set of corrections consists in allowing the production in the s-channel intermediate state of *one* pair of particles with rapidities of the same order of magnitude, both in the central or in the fragmentation region (quasi-multi-Regge kinematics). This implies one replacement among the following in the production amplitudes (1.2) entering the s-channel unitarity relation:

$$\Gamma_{P'P}^{\mathcal{C}(Born)} \longrightarrow \Gamma_{\{f\}P}^{\mathcal{C}(Born)}, \qquad \gamma_{c_i c_i+1}^{G_i(Born)} \longrightarrow \gamma_{c_i c_i+1}^{Q\bar{Q}(Born)}, \qquad \gamma_{c_i c_i+1}^{G_i(Born)} \longrightarrow \gamma_{c_i c_i+1}^{GG(Born)}.$$

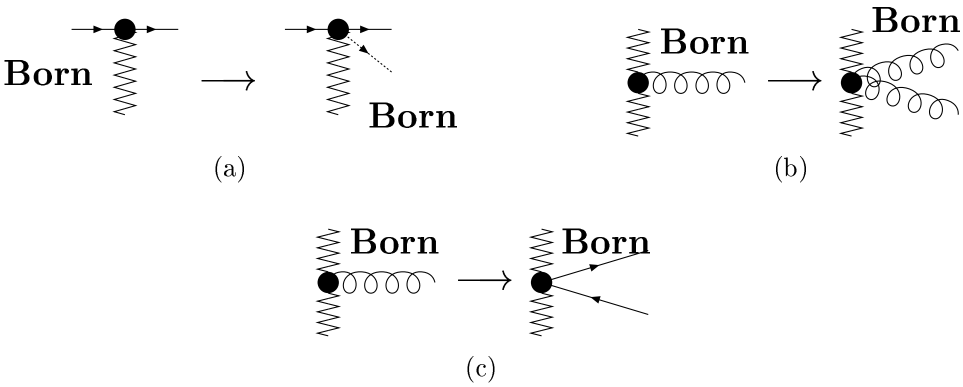

Figure 1.5: Diagrammatical representations of replacements $\Gamma^{\mathcal{C}(Born)}_{P'P} \longrightarrow \Gamma^{\mathcal{C}(Born)}_{\{f\}P}$(a), $\gamma^{G_i(Born)}_{c_ic_i+1} \longrightarrow \gamma^{Q\bar{Q}(Born)}_{c_ic_i+1}$(b) and $\gamma^{G_i(Born)}_{c_ic_i+1} \longrightarrow \gamma^{GG(Born)}_{c_ic_i+1}$(c).

Here $\Gamma_{\{f\}P}$ stands for the production of a state containing an extra-particle in the fragmentation region of the particle P in the scattering off the Reggeon, $\gamma^{Q\bar{Q}(Born)}_{c_ic_i+1}$ and $\gamma^{GG(Born)}_{c_ic_i+1}$ are the effective vertices for the production of a quark-antiquark pair and of a two-gluon pair, respectively, in the collision of two Reggeons. This second set of replacements is shown in Figs. (1.5(a), 1.5(b) and 1.5(c)).

Chapter 2

The next-to-leading order jet vertex for Mueller-Navelet and forward jets

Let us consider the production of Mueller-Navelet jets in proton-proton collisions:

$$p(p_1) + p(p_2) \to \text{jet}(k_{J_1}) + \text{jet}(k_{J_2}) + X \tag{2.1}$$

where the jets are described by their rapidities $y_{1,2}$ and transverse momenta $\vec{k}_{j,1}$ and $\vec{k}_{j,2}$ and are characterized by conditions

$$\vec{k}_{J,1}^2 \sim \vec{k}_{J,2}^2 \gg \Lambda_{QCD}^2 \tag{2.2}$$

and large separation in rapidity; p_1 and p_2 are taken as Sudakov vectors satisfying $p_1^2 = p_2^2 = 0$ and $2(p_1 p_2) = s$ with the following decomposition for a jet in fragmentation region of the proton with momentum p_1,

$$k_{J,1} = x_{J,1} p_1 + \frac{\vec{k}_{J,1}}{x_{J,1}} p_2 + k_{J,1\perp}, \qquad k_{J,1\perp}^2 = -\vec{k}_{J,1}^2 \ . \tag{2.3}$$

In QCD collinear factorization the cross section of the process reads

$$\frac{d\sigma}{dx_{J,1} dx_{J,2} d^2\vec{k}_{J,1} d^2\vec{k}_{J,2}} = \sum_{i,j=q,\bar{q},g} \int_0^1 \int_0^1 dx_1 dx_2 f_i(x_1, \mu) \times f_j(x_2, \mu) \frac{d\hat{\sigma}_{i,j}(x_1 x_2 s, \mu)}{dx_{J,1} dx_{J,2} d^2\vec{k}_{J,1} d^2\vec{k}_{J,2}} \tag{2.4}$$

where the indices i, j specify parton types (quarks $q{=}u, d, s$; antiquarks $\overline{q}{=}\overline{u}, \overline{d}, \overline{s}$ or gluon g), $f_i(x, \mu)$ is the parton distribution function (PDF), $x_{1,2}$ are the longitudinal fractions of the parton involved in the hard subprocess, μ is the factorization scale and $d\hat{\sigma}_{i,j}(x_1 x_2 s, \mu)$ is the partonic cross section for the production of jets.

According to what has been stated in the previous chapter, due to the optical theorem, the cross section is related to the imaginary part of the forward proton-proton scattering amplitude:

$$\hat{\sigma} = \frac{\mathcal{I}m_s \mathcal{A}}{\hat{s}}, \tag{2.5}$$

In the BFKL approach in the kinematical limit $s \gg \vec{k}^2$ the forward amplitude may be presented in D dimension as follows:

$$\begin{aligned} \mathcal{I}m_s \mathcal{A} = \frac{\hat{s}}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \Phi_{P,1}(\vec{q}_1, s_0) \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \Phi_{P,2}(-\vec{q}_2, s_0) \\ \times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{\hat{s}}{s_0} \right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2) \; , \end{aligned} \tag{2.6}$$

where the Green's function obeys the BFKL equation

$$\omega G_\omega(\vec{q}_1, \vec{q}_2) = \delta^{D-2}(\vec{q}_1 - \vec{q}_2) + \int d^{D-2}\vec{q} K(\vec{q}_1, \vec{q}) G_\omega(\vec{q}, \vec{q}_1) \; . \tag{2.7}$$

The energy scale parameter s_0 is arbitrary within NLA accuracy, but in the following we will see that it plays an important role in numerical computations through terms beyond the NLA which unavoidably appear in the NLA prediction. The impact factors Φ_1 and Φ_2 are fundamental ingredients in the description of the inclusive production of the two jets.

2.1 Parton impact factor

Both the kernel of the equation for the Green's function and the parton impact factors can be expressed in terms of the gluon Regge trajectory,

$$j(t) = 1 + \omega(t) \tag{2.8}$$

and the effective vertices for the Reggeon-parton interaction.

To be more specific, let us consider the formulae for the case of forward quark impact factor. Starting with the LO, the quark impact factors are give by

$$\Phi_q^{(0)}(\vec{q}) = \sum_a \int \frac{dM_a^2}{2\pi} \Gamma_{aq}^{(0)}(\vec{q})[\Gamma_{aq}^{(0)}(\vec{q})]^* d\rho_a \ , \tag{2.9}$$

where $\vec{q}$ is the Reggeon transverse momentum, and $\Gamma_{aq}^{(0)}$ denotes the Reggeon-quark vertices in the LO or Born approximation. The sum a is over all intermediate states a which contribute to the $q \to q$ transition. The phase space element $d\rho$ a of a state a, consisting of particles with momenta ℓ_n, is (p_q is initial quark momentum)

$$d\rho_a = (2\pi)^D \delta^{(D)} \left(p_q + q - \sum_{n \in a} \ell_n \right) \prod_{n \in a} \frac{d^{D-1}\ell_n}{(2\pi)^{D-1} 2E_n} \ , \tag{2.10}$$

while the remaining integration in (2.9) is over the squared invariant mass of the state a,

$$M_a^2 = (p_q + q)^2 \ .$$

In the LO the only intermediate state which contributes is a one-quark state, $a = q$. The integration in Eq. (2.9) with the known Reggeon-quark vertices $\Gamma_{qq}^{(0)}$ is trivial and the quark impact factor reads

$$\Phi_q^{(0)}(\vec{q}) = g^2 \frac{\sqrt{N_c^2 - 1}}{2N_c} \ , \tag{2.11}$$

where g is QCD coupling, $\alpha_s = \frac{g^2}{4\pi}$, $N_c = 3$ is the number of QCD colors.

In the NLO the expression (2.9) for the quark impact factor has to be changed in two ways. First, one has to take into account the radiative corrections to the vertices,

$$\Gamma_{qq}^{(0)} \to \Gamma_{qq} = \Gamma_{qq}^{(0)} + \Gamma_{qq}^{(1)} \ .$$

Second, in the sum over $\{a\}$ in (2.9), we have to include more complicated states which appear in the next order of perturbative theory. For the quark impact factor this is a state with an additional gluon, $a = qg$. However, the integral over M_a^2 becomes divergent when an extra gluon appears in the final state. The divergence arises because the gluon may be emitted not only in the fragmentation region of initial quark, but also in the central rapidity region. The contribution of the central region must be subtracted from the impact factor, since it is to be assigned in the BFKL approach to the Green's function. Therefore the result for the forward quark impact factor, see [11], reads

$$\begin{aligned} \Phi_q^{(0)}(\vec{q}\,) = & \left(\frac{s_0}{\vec{q}^{\,2}}\right)^{\omega(-\vec{q}^{\,2})} \sum_{\{a\}} \int \frac{dM_a^2}{2\pi} \Gamma_{aq}(\vec{q}\,)[\Gamma_{aq}(\vec{q}\,)]^* d\rho_a \theta(s_\Lambda - M_a^2) \\ & - \frac{1}{2} \int d^{D-2}k \frac{\vec{q}^{\,2}}{\vec{k}^{2}} \Phi_q^{(0)}(\vec{k}) \mathcal{K}_r^{(0)}(\vec{k}, \vec{q}\,) \ln\left(\frac{s_\Lambda^2}{(\vec{k} - \vec{q}\,)^2 s_0}\right) \ , \end{aligned} \tag{2.12}$$

where the intermediate parameter s_Λ should go to infinity. The second term in the r.h.s. of (2.12) is the subtraction of the gluon emission in the central rapidity region. The dependence on s_Λ vanishes because of the cancellation between the first and second terms. $K_r^{(0)}$ is the part of LO BFKL kernel related to real gluon production

$$K_r^{(0)}(\vec{k}, \vec{q}\,) = \frac{2g^2 N_c}{(2\pi)^{D-1}} \frac{1}{(\vec{k} - \vec{q}\,)^2} \ . \tag{2.13}$$

The factor in (2.12) which involves the Regge trajectory arises from the change of energy scale ($\vec{q}^{\,2} \to s_0$) in the vertices Γ. The trajectory function $\omega(t)$ can be taken here in the one-loop approximation ($t = -\vec{q}^{\,2}$),

$$\omega(t) = \frac{g^2 t}{(2\pi)^{D-1}} \frac{N_c}{2} \int \frac{d^{D-2}k}{\vec{k}^2 (\vec{q} - \vec{k})^2} = -g^2 N_c \frac{\Gamma(1-\varepsilon)\Gamma^2(\varepsilon)}{(4\pi)^{D/2}\Gamma(2\varepsilon)} (\vec{q}^{\,2})^{\varepsilon} \ . \tag{2.14}$$

In Eqs. (2.9) and (2.12) we suppress for shortness the color indices (for the explicit form of the vertices see [10, 11]). The gluon impact factor $\Phi_g^{(0)}(\vec{q})$ is defined similarly. In the gluon case only the single-gluon intermediate state contributes in the LO, $a = g$, which results in

$$\Phi_g^{(0)}(\vec{q}) = \frac{C_A}{C_F}\Phi_q^{(0)}(\vec{q}) \; , \tag{2.15}$$

here $C_A = N_c$ and $C_F = (N_c^2 - 1)/(2N_c)$. Whereas in NLO additional two-gluon, $a = gg$, and quark-antiquark, $a = q\bar{q}$, intermediate states have to be taken into account in the calculation of the gluon impact factor.

2.2 Jet impact factor

Similarly to the parton-parton scattering (2.6) one can represent the resummed jet cross section in the form

$$\begin{aligned} \frac{d\sigma}{dJ_1 dJ_2} &= \frac{1}{(2\pi)^{D-2}} \int \frac{d^{D-2}\vec{q}_1}{\vec{q}_1^{\,2}} \frac{d\Phi_{J,1}(\vec{q}_1, s_0)}{dJ_1} \int \frac{d^{D-2}\vec{q}_2}{\vec{q}_2^{\,2}} \frac{d\Phi_{J,2}(-\vec{q}_2, s_0)}{dJ_2} \\ &\times \int_{\delta - i\infty}^{\delta + i\infty} \frac{d\omega}{2\pi i} \left(\frac{\hat{s}}{s_0}\right)^{\omega} G_\omega(\vec{q}_1, \vec{q}_2) \; , \end{aligned} \tag{2.16}$$

where we introduce jet impact factors differential with respect to the variables parameterizing the jet phase space,

$$dJ_1 \equiv dx_{J,1} d^{D-2}k_{J,1}, \qquad dJ_2 \equiv dx_{J,2} d^{D-2}k_{J,2} \; .$$

Following [7], we consider our process in the frame of a generic and infrared-safe jet algorithm. In practice, this is done by introducing into the integration over the partonic phase space a suitably defined function which identifies the jet momentum with the momentum of one parton or with the sum of the two or more parton momenta when the jet is originated from the a multi-parton intermediate state. In our accuracy the jet can be formed by one parton in LO and by one or two partons when the process is considered in NLO. In the simplest case, the jet momentum is identified with the momentum of the parton in the intermediate state k by the following jet function [31]:

$$S_J^{(2)}(\vec{k}; x) = \delta(x - x_J)\delta^{(D-2)}(\vec{k} - \vec{k}_J) \; . \tag{2.17}$$

In the more complicated case when the jet originates from a state of two partons with momenta k_1 and k_2, we need another function $S_J^{(3)}$, whose explicit form is specific for the chosen jet algorithm. An example of jet selection function in the case of the *cone algorithm* is the following [31]:

$$
\begin{aligned}
S_J^{(3,\mathrm{cone})} \quad & (\vec{k}_2, \vec{k}_1, x\beta_1; x) = S_J^{(2)}(\vec{k}_2; x(1-\beta_1)) \\
& \times\Theta\left([\Delta y^2 + \Delta\phi^2] - \left[\frac{|\vec{k}_1| + |\vec{k}_2|}{\max(|\vec{k}_1|, |\vec{k}_2|)} R_{\mathrm{cone}}\right]^2\right) \\
& + S_J^{(2)}(\vec{k}_1; x\beta_1)\Theta\left([\Delta y^2 + \Delta\phi^2] - \left[\frac{|\vec{k}_1| + |\vec{k}_2|}{\max(|\vec{k}_1|, |\vec{k}_2|)} R_{\mathrm{cone}}\right]^2\right) \\
& + S_J^{(2)}(\vec{k}_1 + \vec{k}_2; x)\Theta\left(\left[\frac{|\vec{k}_1| + |\vec{k}_2|}{\max(|\vec{k}_1|, |\vec{k}_2|)} R_{\mathrm{cone}}\right]^2 - [\Delta y^2 + \Delta\phi^2]\right) ,
\end{aligned}
\tag{2.18}
$$

where the Sudakov decomposition of the parton momenta

$$
k_1 = x\beta_1 p_1 + \frac{\vec{k}_1^2}{x\beta_1 s} p_2 + k_{1_\perp}, \qquad k_1^2 = 0, \tag{2.19}
$$

$$
k_2 = x\beta_2 p_1 + \frac{\vec{k}_2^2}{x\beta_2 s} p_2 + k_{2_\perp}, \qquad k_2^2 = 0 \tag{2.20}
$$

is used, with $\beta_1 + \beta_2 = 1$ and $\vec{k}_1 + \vec{k}_2 = \vec{q}$, owing to momentum conservation in the partonic subprocess. R_{cone} in (2.19) is the cone-size parameter, Δy and $\Delta\phi$ are the difference of rapidity and azimuthal angle in the two parton state, respectively:

$$
\Delta y = \ln\left(\frac{1-\beta_1}{\beta_1}\frac{|\vec{k}_1|}{|\vec{k}_2|}\right), \qquad \Delta\phi = \arccos\frac{\vec{k}_1 \cdot \vec{k}_2}{\sqrt{\vec{k}_1^2 \vec{k}_2^2}} \,. \tag{2.21}
$$

The three terms in $S_J^{(3,\mathrm{cone})}$ represent the case in which the jet is formed by the parton k_2 or the parton k_1 or both, respectively.

In the generic case, the following relations for the jet function must be fulfilled in order the jet algorithm be infrared safe:

$$
\begin{aligned}
S_J^{(3)}(\vec{k}_2, \vec{k}_1, x\beta_1; x) &\to S_J^{(2)}(\vec{k}_2; x), && \vec{k}_1 \to 0, \beta_1 \to 0, \\
S_J^{(3)}(\vec{k}_2, \vec{k}_1, x\beta_1; x) &\to S_J^{(2)}(\vec{k}_1 + \vec{k}_2; x), && \vec{k}_1\beta_1 \to \vec{k}_2\beta_1, \\
S_J^{(3)}(\vec{k}_2, \vec{k}_1, x\beta_1; x) &\to S_J^{(2)}(\vec{k}_2; x(1-\beta_1)), && \vec{k}_1 \to 0, \\
S_J^{(3)}(\vec{k}_2, \vec{k}_1, x\beta_1; x) &\to S_J^{(2)}(\vec{k}_1; x\beta_1), && \vec{k}_2 \to 0\ .
\end{aligned} \tag{2.22}
$$

Such reduction of $S_J^{(3)} \to S_J^{(2)}$ is required in order that the singular contributions generated by the real emission be proportional to the lowest order cross section. These contributions are canceled with the soft and collinear singularities arising from the virtual corrections and the collinear counterterms coming from the PDFs' renormalization. Besides, we assume that the jet selection function $S_J^{(3)}$ is symmetric under the exchange of the final state parton kinematic variables, $\beta_1 \leftrightarrow \beta_2$ and $\vec{k}_1 \leftrightarrow \vec{k}_2$,

$$
S_J^{(3,\mathrm{cone})}(\vec{k}_2, \vec{k}_1, x\beta_1; x) = S_J^{(3,\mathrm{cone})}(\vec{k}_1, \vec{k}_2, x\beta_2; x)\ . \tag{2.23}
$$

The collinear counterterms appear due to the replacement of the bare PDFs by the renormalized physical quantities which obey DGLAP evolution equations, in the factorization $\overline{\mathrm{MS}}$ scheme:

$$
\begin{aligned}
f_q(x) &= f_q(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\Bigg[P_{qq}(z) f_q\left(\frac{x}{z},\mu_F\right) \\
&\quad + P_{qg}(z) f_g\left(\frac{x}{z},\mu_F\right)\Bigg] \\
f_g(x) &= f_g(x,\mu_F) - \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_F^2}{\mu^2}\right)\int_x^1 \frac{dz}{z}\Bigg[P_{gq}(z) f_q\left(\frac{x}{z},\mu_F\right) \\
&\quad + P_{gg}(z) f_g\left(\frac{x}{z},\mu_F\right)\Bigg]\ ,
\end{aligned} \tag{2.24}
$$

where $\frac{1}{\hat{\varepsilon}} = \frac{1}{\varepsilon} + \gamma_E - \ln(4\pi) \approx \frac{\Gamma(1-\varepsilon)}{\varepsilon(4\pi)^\varepsilon}$ and the DGLAP splitting functions are:

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z}\right)_+ = C_F \left[\frac{1+z^2}{(1-z)_+} + \frac{3}{2}\delta(1-z)\right], \tag{2.25}$$

$$P_{qg}(z) = T_R \left[z^2 + (1-z)^2\right], \qquad \text{with } T_R = \frac{1}{2}, \tag{2.26}$$

$$P_{gg}(z) = 2C_A \left(\frac{z}{(1-z)_+} + \frac{(1-z)}{z} + z(1-z)\right) + \frac{(11C_A - 4N_F T_R)}{6}\delta(1-z), \tag{2.27}$$

$$P_{gq}(z) = C_F \frac{[1+(1-z)^2]}{z}; \tag{2.28}$$

here the plus-prescription is defined by

$$\int_a^1 dx \frac{F(x)}{(1-x)_+} = \int_a^1 dx \frac{F(x)-F(1)}{1-x} - \int_0^a dx \frac{F(1)}{1-x} \tag{2.29}$$

The other counterterm is related with QCD charge renormalization, in the $\overline{\text{MS}}$ scheme:

$$\alpha_s = \alpha_s(\mu_R)\left[1 + \frac{\alpha_s(\mu_R)}{4\pi}\left(\frac{11C_A}{3} - \frac{2N_F}{3}\right)\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\right]. \tag{2.30}$$

Having the results for the lowest order parton impact factors (2.11) and (2.15), we get the jet impact factor at the LO level as

$$\frac{d\Phi_J^{(0)}(\vec{q})}{dJ} = g^2 \frac{\sqrt{N_c^2-1}}{2N_c} \int_0^1 dx \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right) S_J^{(2)}(\vec{q};x), \tag{2.31}$$

given as the sum of the gluon and all possible quark and antiquark PDFs contributions. Substituting here the bare QCD coupling and bare PDFs by the renormalized ones, we obtain the following expressions for the counterterms:

$$\begin{aligned} \frac{d\Phi_J(\vec{q})|_{\text{charge c.t.}}}{dJ} &= \frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}} + \ln\frac{\mu_R^2}{\mu^2}\right)\left(\frac{11C_A}{6} - \frac{N_F}{3}\right)\Phi_q^{(0)} \\ &\times \int_0^1 dx \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right) S_J^{(2)}(\vec{q};x) \end{aligned} \tag{2.32}$$

for the charge renormalization, and

$$\frac{d\Phi_J(\vec{q})|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}}+\ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 dx S_J^{(2)}(\vec{q};x)\int_x^1\frac{dz}{z}$$
$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(z)f_a\left(\frac{x}{z}\right)+P_{qg}(z)f_g\left(\frac{x}{z}\right)\right)\right. \tag{2.33}$$
$$\left.+\frac{C_A}{C_F}\left(P_{gg}(z)f_g\left(\frac{x}{z}\right)+P_{gq}(z)\sum_{a=q,\bar{q}}f_a\left(\frac{x}{z}\right)\right)\right]$$

for the collinear counterterm. The latter can be rewritten in the form

$$\frac{d\Phi_J(\vec{q})|_{\text{collinear c.t.}}}{dJ} = -\frac{\alpha_s}{2\pi}\left(\frac{1}{\hat{\varepsilon}}+\ln\frac{\mu_F^2}{\mu^2}\right)\Phi_q^{(0)}\int_0^1 d\beta\int_0^1 dx S_J^{(2)}(\vec{q};\beta x)$$
$$\times\left[\sum_{a=q,\bar{q}}\left(P_{qq}(\beta)f_a(x)+P_{qg}(\beta)f_g(x)\right)\right. \tag{2.34}$$
$$\left.+\frac{C_A}{C_F}\left(P_{gg}(\beta)f_g(x)+P_{gq}(\beta)\sum_{a=q,\bar{q}}f_a(x)\right)\right] .$$

Besides, we present the expression for the BFKL counterterm which, in accordance to the second line of Eq. (2.12), provides the subtraction of the gluon radiation in the central rapidity region:

$$\frac{d\Phi_J(\vec{q})|_{\text{BFKL c.t.}}}{dJ} = -\Phi_q^{(0)}\frac{C_A g^2}{(2\pi)^{D-1}}\int_0^1 dx\left(\frac{C_A}{C_F}f_g(x)+\sum_{a=q,\bar{q}}f_a(x)\right)$$
$$\times\int d^{D-2}\vec{k}\frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2}\ln\left(\frac{s_\Lambda^2}{s_0\,\vec{k}^2}\right)S_J^{(2)}(\vec{q}-\vec{k};x) . \tag{2.35}$$

Now we have all the necessary ingredients to perform our calculation of the NLO corrections to the jet impact factor. As a starting point for our consideration we will use the results of [10, 11] for the partonic amplitudes obtained in the calculation of partonic impact factors, introducing there the appropriate jet functions: $S_J^{(2)}$ for the amplitudes with one-parton state in the case of one-loop virtual corrections and $S_J^{(3)}$ for the cases with two partons in the final state (real emission), in order to define the corresponding contribution to jet cross sections.

For shortness we will present intermediate results for V structures defined always as

$$\frac{d\Phi_J^{(1)}(\vec{q}\,)}{dJ} \equiv \frac{\alpha_s}{2\pi}\Phi_q^{(0)}V(\vec{q}\,)\ . \tag{2.36}$$

We will consider separately the subprocesses initiated by the quark and the gluon PDFs and denote

$$V = V_q + V_g. \tag{2.37}$$

2.3 NLA jet impact factor: the quark contribution

We start with the case of incoming quark.

2.3.1 Virtual correction

Virtual corrections are the same as in the case of the inclusive quark impact factor [10, 11, 12]:

$$\begin{aligned} V_q^{(V)}(\vec{q}\,) = & -\frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\vec{q}^{\,2})^\varepsilon\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)S_J^{(2)}(\vec{q};x) \\ & \times C_F\left\{\left(\frac{2}{\varepsilon}-\frac{4}{1+2\varepsilon}+1\right) - N_F\frac{1+\varepsilon}{(1+2\varepsilon)(3+2\varepsilon)}\right. \\ & + C_A\left(\ln\frac{s_0}{\vec{q}^{\,2}}+\psi(1-\varepsilon)-2\psi(\varepsilon)+\psi(1)\right. \\ & \left.\left. + \frac{1}{4(1+2\varepsilon)(3+2\varepsilon)} - \frac{2}{\varepsilon(1+2\varepsilon)} - \frac{7}{4(1+2\varepsilon)} - \frac{1}{2}\right)\right\}. \end{aligned} \tag{2.38}$$

Here and in what follows we put the arbitrary scale of dimensional regularization equal to unity, $\mu = 1$. We expand (2.38) in ε and get

$$\begin{aligned} V_q^{(V)}(\vec{q}\,) = & -\frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\vec{q}^{\,2})^\varepsilon\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x)S_J^{(2)}(\vec{q};x) \\ & \times \left[C_F\left(\frac{2}{\varepsilon}-3\right) - \frac{N_F}{3} + C_A\left(\ln\frac{s_0}{\vec{q}^{\,2}}+\frac{11}{6}\right)\right. \\ & \left. + \varepsilon\left\{8C_F + \frac{5N_F}{9} - C_A\left(\frac{85}{18}+\frac{\pi^2}{2}\right)\right\}\right] + \mathcal{O}(\varepsilon)\ . \end{aligned} \tag{2.39}$$

2.3.2 Real correction

For the incoming quark case, real corrections originate from the quark-gluon intermediate state. We denote the momentum of the gluon by k, then the momentum of the quark is $q-k$; the longitudinal fraction of the gluon momentum is denoted by βx. Thus, the real contribution has the form [10, 11, 12, 32]

$$
\begin{aligned}
V_q^{(V)}(\vec{q}) = \frac{1}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 d\beta \mathcal{P}_{gq}(\varepsilon,\beta) \\
\times \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{q}-\vec{k})^2(\vec{k}-\beta\vec{q})^2}\left\{C_F\beta^2(\vec{q}-\vec{k})^2 + C_A(1-\beta)\vec{k}\cdot(\vec{k}-\beta\vec{q})\right\} \\
\times S_J^{(3)}\left(\vec{q}-\vec{k},\vec{k},x\beta,x\right),
\end{aligned}
\tag{2.40}
$$

where

$$
\beta_0 = \frac{\vec{k}^2}{s_\Lambda}, \qquad \mathcal{P}_{gq}(\varepsilon,\beta) = \frac{1+(1-\beta)^2+\varepsilon\beta^2}{\beta}\ .
$$

The low limit in the β-integration appears due to the restriction on the invariant mass of intermediate state, which enters the definition (2.12) of NLO impact factor. Since

$$
M_{qg}^2 = \frac{\vec{k}^2}{\beta} + \frac{(\vec{q}-\vec{k})^2}{1-\beta} - \vec{q}^{\,2}, \qquad M_{qg}^2 \le s_\Lambda\ ,
$$

and assuming $s_\Lambda \to \infty$, we obtain that $\beta \ge \beta_0$.

We consider separately the term proportional to C_F and to C_A. The C_F-term is not singular for $\beta \to 0$, therefore the limit $s_\Lambda \to \infty$, or $\beta_0 \to 0$, can be safely taken. We get

$$
\begin{aligned}
V_q^{(R)(C_F)}(\vec{q}) = \frac{C_F}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_0^1 d\beta \mathcal{P}_{gq}(\varepsilon,\beta) \\
\times \frac{\vec{q}^{\,2}\beta^2}{\vec{k}^2(\vec{k}-\beta\vec{q})^2} S_J^{(3)}\left(\vec{q}-\vec{k},\vec{k},x\beta,x\right).
\end{aligned}
\tag{2.41}
$$

In order to isolate all divergences, it is convenient to perform the change of variable $\vec{k} = \beta\vec{l}$ and to present the integral in the form

$$
\begin{aligned}
V_q^{(R)(C_F)}(\vec{q}) &= \frac{C_F}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int_0^1 d\beta \mathcal{P}_{gq}(\varepsilon,\beta)\beta^{2\varepsilon} \qquad (2.42) \\
&\times \int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{l}^{\,2}+(\vec{l}-\vec{q})^2}\left[\frac{1}{(\vec{l}-\vec{q})^2}+\frac{1}{\vec{l}^{\,2}}\right] S_J^{(3)}(\vec{q}-\beta\vec{l},\beta\vec{l},x\beta;x)\ .
\end{aligned}
$$

The soft divergence appears for $\beta \to 0$; in this region we can introduce the counterterm

$$V_q^{(R)(C_F,\,\mathrm{soft})}(\vec{q}) = \frac{C_F}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int_0^1 d\beta \frac{2}{\beta^{1-2\varepsilon}} \times \int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{l}^{\,2}+(\vec{l}-\vec{q})^2}\left[\frac{1}{(\vec{l}-\vec{q})^2}+\frac{1}{\vec{l}^{\,2}}\right] S_J^{(2)}(\vec{q};x)\ , \tag{2.43}$$

which equals

$$V_q^{(R)(C_F,\,\mathrm{soft})}(\vec{q}) = \frac{2C_F}{\varepsilon}\frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\vec{q}^{\,2})^\varepsilon \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) S_J^{(2)}(\vec{q};x)\ . \tag{2.44}$$

Collinear divergences arise for $\vec{l}-\vec{q}=0$ and for $\vec{l}=0$; in these regions we can isolate the two following counterterms:

$$V_q^{(R)(C_F,\,\mathrm{coll}_1)}(\vec{q}) = \frac{C_F}{(4\pi)^\varepsilon}\int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}(\vec{l}-\vec{q})^2}\Theta(\Lambda^2-(\vec{l}-\vec{q})^2) \tag{2.45}$$

$$\times \int_0^1 d\beta\, \beta^{2\varepsilon}\left[\mathcal{P}_{gq}(\varepsilon,\beta)-\frac{2}{\beta}\right]\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) S_J^{(2)}(\vec{q};x)\ ,$$

$$V_q^{(R)(C_F,\,\mathrm{coll}_2)}(\vec{q}) = \frac{C_F}{(4\pi)^\varepsilon}\int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}\vec{l}^{\,2}}\Theta(\Lambda^2-\vec{l}^{\,2})\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \times \int_0^1 d\beta\, \beta^{2\varepsilon}\left[S_J^{(2)}(\vec{q};x(1-\beta))\mathcal{P}_{gq}(\varepsilon,\beta)-\frac{2}{\beta}S_J^{(2)}(\vec{q};x)\right]. \tag{2.46}$$

In both these expressions we have introduced an arbitrary cutoff parameter Λ and subtracted the soft divergence. After a simple calculation we obtain

$$V_q^{(R)(C_F,\,\mathrm{coll}_1)}(\vec{q}) = \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) S_J^{(2)}(\vec{q};x) \times C_F\left[-\frac{3}{2}+4\varepsilon\right]+\mathcal{O}(\varepsilon)\ . \tag{2.47}$$

The term $V_q^{(R)(C_F,\,\mathrm{coll}_2)}$can be rewritten in the following form:

$$\begin{aligned} V^{(R)(C_F,\mathrm{coll}_2)}(\vec{q}) &= \frac{\Gamma[1-\varepsilon]}{\varepsilon\,(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\left(\Lambda^2\right)^\varepsilon \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \\ &\times \left\{-\frac{3}{2}C_F S_J^{(2)}(\vec{q};x) + \int_0^1 d\beta\left[P_{qq}(\beta) + 2\varepsilon(1+\beta^2)\right.\right. \\ &\times \left.\left(\frac{\ln(1-\beta)}{1-\beta}\right)_+ C_F + \varepsilon\, C_F(1-\beta)\right] \\ &\times \left. S_J^{(2)}(\vec{q};x\beta)\right\} + \mathcal{O}(\varepsilon)\;, \end{aligned} \tag{2.48}$$

where we performed the change of variable $\beta \to 1-\beta$, used the plus-prescription (2.29) and the expansion

$$(1-\beta)^{2\varepsilon-1} = \frac{1}{2\varepsilon}\delta(1-\beta) + \frac{1}{(1-\beta)_+} + 2\varepsilon\left(\frac{\ln(1-\beta)}{1-\beta}\right)_+ + \mathcal{O}(\varepsilon^2)\;.$$

Finally, we can define the term

$$V_q^{(R)(C_F,\,\mathrm{finite})} = V_q^{(R)(C_F)} - V_q^{(R)(C_F,\mathrm{soft})} - V_q^{(R)(C_F,\,\mathrm{coll}_1)} - V_q^{(R)(C_F,\,\mathrm{coll}_2)}\;, \tag{2.49}$$

which can be calculated at $\varepsilon = 0$. We remark that $V_q^{(R)(C_F,\,\mathrm{finite})}$ and $V_q^{(R)(C_F,\,\mathrm{coll}_{1,2})}$ depend on the cutoff Λ, but in the total expression $V_q^{(R)(C_F,)}$ this dependence disappears. The part proportional to C_A in the r.h.s. of Eq. (2.40) reads

$$\begin{aligned} V_q^{(R)(C_A)}(\vec{q}) = \frac{1}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 d\beta \mathcal{P}_{gq}(\varepsilon,\beta) \\ \times\, \vec{q}^{\,2} C_A \frac{(1-\beta)\vec{k}\cdot(\vec{k}-\beta\vec{q})}{\vec{k}^2(\vec{q}-\vec{k})^2(\vec{k}-\beta\vec{q})^2} S_J^{(3)}\left(\vec{q}-\vec{k},\vec{k},x\beta;x\right)\;. \end{aligned} \tag{2.50}$$

The collinear singularity appears at $\vec{k}-\vec{q} \to 0$; in this region we can introduce the counterterm

$$V_q^{(R)(C_A,\,\mathrm{coll})}(\vec q) = \frac{C_A}{(4\pi)^\varepsilon}\int \frac{d^{D-2}\vec k}{\pi^{1+\varepsilon}(\vec q-\vec k)^2}\Theta\left(\Lambda^2-(\vec k-\vec q\,)^2\right) \times \int_0^1 dx\sum_{a=q,\bar q} f_a(x)\int_0^1 d\beta \mathcal{P}_{gq}(\varepsilon,\beta) S_J^{(2)}(\vec q; x\beta)$$
$$= \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon\int_0^1 dx\sum_{a=q,\bar q} f_a(x) \times \int_0^1 d\beta\left[\frac{C_A}{C_F}P_{gq}(\beta)+\varepsilon\, C_A\beta\right] S_J^{(2)}(\vec q; x\beta)+\mathcal{O}(\varepsilon)\,, \tag{2.51}$$

where β_0 has been set equal to zero since the expression is finite in the $\beta\to 0$ limit and, again, the cutoff Λ was introduced. Another singularity appears when $\beta\to 0$, actually at any value of gluon transverse momentum $\vec k$. In this region $S_J^{(3)}\left(\vec q-\vec k,\vec k, x\beta; x\right)\to S_J^{(2)}(\vec q-\vec k; x)$ and it is convenient to introduce the counterterm

$$V_q^{(R)(C_A,\,\mathrm{soft})}(\vec q) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\sum_{a=q,\bar q} f_a(x)\int\frac{d^{D-2}\vec k}{\pi^{1+\varepsilon}}\int_{\beta_0}^1 d\beta\frac{2}{\beta} \times \vec q^{\,2}\frac{(1-\beta)\vec k\cdot(\vec k-\beta\vec q)}{\vec k^2(\vec q-\vec k)^2(\vec k-\beta\vec q)^2} S_J^{(2)}\left(\vec q-\vec k; x\right)$$
$$= \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\sum_{a=q,\bar q} f_a(x)\int\frac{d^{D-2}\vec k}{\pi^{1+\varepsilon}}\int_{\beta_0}^1 d\beta\frac{2}{\beta} \times \frac{\vec q^{\,2}\Theta[(1-\beta)|\vec k|-\beta|\vec q-\vec k|]}{\vec k^2(\vec q-\vec k)^2} S_J^{(2)}\left(\vec q-\vec k; x\right)\,, \tag{2.52}$$

where the averaging over the relative angle between the vectors $\vec k$ and $\vec q-\vec k$ has been performed. The integration over β gives the following result for the counterterm:

$$V_q^{(R)(C_A,\,\mathrm{soft})}(\vec q) = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\sum_{a=q,\bar q} f_a(x)\int\frac{d^{D-2}\vec k}{\pi^{1+\varepsilon}}\frac{\vec q^{\,2}}{\vec k^2(\vec q-\vec k)^2} \times \ln\frac{s_\Lambda^2}{\vec k^2(|\vec k|+|\vec q-\vec k|)^2} S_J^{(2)}\left(\vec q-\vec k; x\right)\,. \tag{2.53}$$

The finite part of the real corrections proportional to C_A is therefore defined by

$$V_q^{(R)(C_A,\,\mathrm{finite})} = V_q^{(R)(C_A)} - V_q^{(R)(C_A,\,\mathrm{coll})} - V_q^{(R)(C_A,\,\mathrm{soft})}\,. \tag{2.54}$$

When the quark part of BFKL counterterm, given in (2.35),

$$V_q^{(C)}(\vec{q}) = -\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \ln\left(\frac{s_\Lambda^2}{s_0\vec{k}^2}\right)\frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{q}-\vec{k})^2} S_J^{(2)}(\vec{q}-\vec{k};x) \ , \tag{2.55}$$

is combined with $V_q^{(R)(C_A,\text{soft})}$ given in (2.53), we see that the dependence on s_Λ disappears, as expected, and we get

$$\begin{aligned} V_q^{(R)(C_A,\text{soft})}(\vec{q}) + V_q^{(C)}(\vec{q}) &= \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \\ \times \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} &\ln\left(\frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2}\right) S_J^{(2)}(\vec{q}-\vec{k};x) \ . \end{aligned} \tag{2.56}$$

2.3.3 Final result for the quark in the initial state

We collect first the contributions given in (2.39), (2.44), (2.47), (2.48) and (2.52):

$$\begin{aligned} V_q^{(1)}(\vec{q}) &\equiv (V_q^{(V)} + V_q^{(R)(C_F,\text{soft})} + V_q^{(R)(C_F,\text{coll}_1)} + V_q^{(R)(C_F,\text{coll}_2)} + V_q^{(R)(C_A,\text{coll})})(\vec{q}) \\ &= \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \\ &\left\{\left[(\vec{q}^{\,2})^\varepsilon\left(\frac{N_F}{3} - C_A\ln\left(\frac{s_0}{\vec{q}^{\,2}}\right) - \frac{11C_A}{6}\right) + \varepsilon\left(C_A\left(\frac{85}{18}+\frac{\pi^2}{2}\right) - \frac{5}{9}N_F \right.\right.\right. \\ &\left.\left. + C_F\left(3\ln\frac{\vec{q}^{\,2}}{\Lambda^2} - 4\right)\right)\right] S_J^{(2)}(\vec{q};x) \\ &+ (\Lambda^2)^\varepsilon \int_0^1 d\beta \left[P_{qq}(\beta) + \frac{C_A}{C_F}P_{gq}(\beta)\right] S_J^{(2)}(\vec{q};x\beta) \\ &+ \varepsilon\int_0^1 d\beta\left[2(1+\beta^2)\left(\frac{\ln(1-\beta)}{1-\beta}\right)_+ C_F + C_F(1-\beta) + C_A\beta\right] \\ &\left.\times S_J^{(2)}(\vec{q};x\beta)\right\} + \mathcal{O}(\varepsilon) \ . \end{aligned} \tag{2.57}$$

Then, we collect the finite contributions, given in Eqs. (2.49) and (2.54), transforming them to the form used in [7]:

$$
\begin{aligned}
V_q^{(2)}(\vec{q}) &\equiv (V_q^{(R)(C_F,\,\text{finite})} + V_q^{(R)(C_A,\,\text{finite})})(\vec{q}) \\
= \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \Bigg[C_F \int_0^1 d\beta \frac{1}{(1-\beta)_+}(1+\beta^2) \int \frac{d^2\vec{l}}{\pi\vec{l}^{\,2}} \Bigg[\frac{\vec{q}^{\,2}}{\vec{l}^{\,2}+(\vec{q}-\vec{l})^2} \\
\times \big\{ S_J^{(3)}(\vec{q}-(1-\beta)\vec{l},(1-\beta)\vec{l},x(1-\beta);x) \\
+ S_J^{(3)}(\vec{q}\beta+(1-\beta)\vec{l},(1-\beta)(\vec{q}-\vec{l}),x(1-\beta);x) \big\} \\
- \Theta(\Lambda^2-\vec{l}^{\,2}) \big\{ S_J^{(2)}(\vec{q};x\beta) + S_J^{(2)}(\vec{q};x) \big\} \Bigg] \qquad (2.58) \\
+ C_A \int \frac{d^2\vec{k}}{\pi\vec{k}^2} \int_0^1 d\beta \Bigg\{ \frac{1+(1-\beta)^2}{\beta} \Bigg[\frac{\vec{q}^{\,2}(1-\beta)(\vec{q}-\vec{\beta})\cdot(\vec{q}(1-\beta)-\vec{k})}{(\vec{q}-\vec{k})^2(\vec{q}(1-\beta)-\vec{k})^2} \\
S_J^{(3)}(\vec{k},\vec{q}-\vec{k},x\beta;x) - \Theta(\Lambda^2-\vec{k}^2) S_J^{(2)}(\vec{q};x\beta) \Bigg] \\
- \frac{2\vec{q}^{\,2}\Theta[(1-\beta)|\vec{q}-\vec{k}|-\beta|\vec{k}|]}{\beta(\vec{q}-\vec{k})^2} S_J^{(2)}(\vec{k};x) \Bigg\} \Bigg] + \mathcal{O}(\varepsilon)\,.
\end{aligned}
$$

Besides, we define

$$
V_q^{(3)}(\vec{q}) \equiv (V_q^{(R)(C_A,\,\text{soft})} + V_q^{(C)})(\vec{q})\,,
$$

given in Eq. (2.56) Another contribution originates from the collinear and charge renormalization counterterms, see Eqs. (2.32) and (2.34),

$$
\begin{aligned}
V_q^{(4)}(\vec{q}) = \frac{\Gamma[1-\beta]}{\varepsilon(4\pi)^\varepsilon} \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \Bigg[(\mu_R^2)^\varepsilon \left(\frac{11C_A}{6} - \frac{N_F}{3} \right) S_J^{(2)}(\vec{q};x) \\
- (\mu_F^2)^\varepsilon \int_0^1 d\beta \Bigg[P_{qq}(\beta) + \frac{C_A}{C_F} P_{gq}(\beta) \Bigg] S_J^{(2)}(\vec{q};x\beta) \Bigg]\,. \qquad (2.59)
\end{aligned}
$$

Finally, the quark part of the jet impact factor is given by the sum of the above four contributions and can be presented as the sum of two terms:

$$
\begin{aligned}
V_q^{(I)}(\vec{q}) = \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \Bigg[\frac{C_A}{(4\pi)^\varepsilon} \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} \ln \frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2} \\
\times S_J^{(2)}(\vec{k};x) - C_A \ln\left(\frac{s_0}{\vec{q}^{\,2}}\right) (\vec{q}^{\,2})^\varepsilon \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} S_J^{(2)}(\vec{q};x) \Bigg] \qquad (2.60)
\end{aligned}
$$

and

$$V_q^{(II)}(\vec{q}) = V_q^{(2)}(\vec{q}) + \int_0^1 dx \sum_{a=q,\bar{q}} f_a(x) \Bigg\{ \Bigg[\left(\frac{N_F}{3} - \frac{11 C_A}{6} \right) \ln \frac{\vec{q}^{\,2}}{\mu_R^2}$$
$$+ C_A \left(\frac{85}{18} + \frac{\pi^2}{2} \right) - \frac{5}{9} N_F + C_F \left(3 \ln \frac{\vec{q}^{\,2}}{\Lambda^2} - 4 \right) \Bigg] S_J^{(2)}(\vec{q}; x)$$
$$+ \int_0^1 d\beta \left[P_{qq}(\beta) + \frac{C_A}{C_F} P_{gq}(\beta) \right] ln \frac{\Lambda^2}{\mu_F^2} S_J^{(2)}(\vec{q}; x\beta) \tag{2.61}$$
$$+ \int_0^1 d\beta \left[2 \left(\frac{\ln(1-\beta)}{1-\beta} \right)_+ (1+\beta^2) C_F + C_F(1-\beta) + C_A \beta \right] S_J^{(2)}(\vec{q}; x\beta) \Bigg\} \, .$$

2.4 NLA jet impact factor: the gluon contribution

We consider now the case of incoming gluon.

2.4.1 Vitual corrections

Virtual corrections are the same as in the case of the inclusive gluon impact factor [10, 11, 12]:

$$V_g^{(V)}(\vec{q}) = -\frac{\Gamma[1-\varepsilon]}{\varepsilon (4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} (\vec{q}^{\,2})^\varepsilon \int_0^1 dx \frac{C_A}{C_F} f_g(x) S_J^{(2)}(\vec{q}; x)$$
$$\times \left[C_A \ln \left(\frac{s_0}{\vec{q}^{\,2}} \right) + C_A \left(\frac{2}{\varepsilon} - \frac{11 + 9\varepsilon}{2(1+2\varepsilon)(3+2\varepsilon)} \right. \right.$$
$$\left. + \frac{N_F}{C_A} \frac{(1+\varepsilon)(2+\varepsilon) - 1}{(1+\varepsilon)(1+2\varepsilon)(3+2\varepsilon)} + \psi(1) + \psi(1-\varepsilon) - 2\psi(1+\varepsilon) \right) \tag{2.62}$$
$$\left. + C_A \frac{\varepsilon}{(1+\varepsilon)(1+2\varepsilon)(3+2\varepsilon)} \left(1 + \varepsilon - \frac{N_F}{C_A} \right) \frac{1}{(1+\varepsilon)} \right] \, .$$

The ε-expansion has the form

$$V_g^{(V)}(\vec{q}) = -\frac{\Gamma[1-\varepsilon]}{\varepsilon (4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} (\vec{q}^{\,2})^\varepsilon \int_0^1 dx \frac{C_A}{C_F} f_g(x) S_J^{(2)}(\vec{q}; x)$$
$$\times \left[C_A \left(\ln \left(\frac{s_0}{\vec{q}^{\,2}} \right) + \frac{2}{\varepsilon} - \frac{11}{6} \right) + \frac{N_F}{3} \right. \tag{2.63}$$
$$\left. + \varepsilon \left\{ C_A \left(\frac{67}{18} - \frac{\pi^2}{2} \right) - \frac{5}{9} N_F \right\} \right] + \mathcal{O}(\varepsilon) \, .$$

2.4.2 Real corrections: $q\bar{q}$ intermediate state

In the NLO gluon impact factor real corrections come from intermediate states of two particles, which can be quark-antiquark or gluon-gluon. In this Subsection we consider the former. The real contribution from the quark-antiquark case is [10, 11, 12, 32]:

$$V_g^{(R_{q\bar{q}})}(\vec{q}) = \frac{N_F}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int_0^1 d\beta \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2}$$
$$\times T_R \mathcal{P}_{qg}(\varepsilon,\beta)\left[\frac{C_F}{C_A} + \frac{\beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})}{(\vec{k}-\beta\vec{q})^2}\right] S_J^{(3)}(\vec{q}-\vec{k},\vec{k},x\beta;x)\ , \tag{2.64}$$

with

$$\mathcal{P}_{qg}(\varepsilon,\beta) = 1 - \frac{2\beta(1-\beta)}{1+\varepsilon}\ . \tag{2.65}$$

Below we discuss separately the first and the second contributions in the r.h.s. of Eq.(2.64), which we denote $V_g^{(R_{q\bar{q}})(C_F)}$ and $V_g^{(R_{q\bar{q}})(C_A)}$. The first contribution is

$$\begin{aligned} V_g^{(R_{q\bar{q}})(C_F)}(\vec{q}) &= \frac{N_F}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int_0^1 d\beta \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} \\ &\times T_R \mathcal{P}_{qg}(\varepsilon,\beta)\frac{C_F}{C_A} S_J^{(3)}(\vec{q}-\vec{k},\vec{k},x\beta;x) \\ &= \frac{N_F}{(4\pi)^\varepsilon}\int_0^1 dx f_g(x) \int_0^1 d\beta T_R \mathcal{P}_{qg}(\varepsilon,\beta) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \\ &\times \frac{\vec{q}^{\,2}}{\vec{k}^2+(\vec{q}-\vec{k})^2}\left[\frac{1}{\vec{k}^2} + \frac{1}{(\vec{q}-\vec{k})^2}\right] S_J^{(3)}(\vec{q}-\vec{k},\vec{k},x\beta;x)\ . \end{aligned} \tag{2.66}$$

Here we have collinear divergences for $\vec{k}=0$ and $\vec{q}-\vec{k}=0$. The contribution in these kinematical regions is the same, as can be easily seen after the changes of variables $\vec{k}\to\vec{q}-\vec{k}$ and $\beta\to 1-\beta$, since $\mathcal{P}_{qg}(\varepsilon,\beta) = \mathcal{P}_{qg}(\varepsilon,1-\beta)$ and taking into account the property (2.23) that the $S_J^{(3)}$ jet selection function has to possess. Therefore we can write

$$\begin{aligned} V_g^{(R_{q\bar{q}})(C_F)}(\vec{q}) &= \frac{2N_F}{(4\pi)^\varepsilon}\int_0^1 dx f_g(x) \int_0^1 d\beta T_R \mathcal{P}_{qg}(\varepsilon,\beta) \\ &\times \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}\vec{k}^2} \frac{\vec{q}^{\,2}}{\vec{k}^2+(\vec{q}-\vec{k})^2} S_J^{(3)}(\vec{k},\vec{q}-\vec{k},x\beta;x) \end{aligned} \tag{2.67}$$

and isolate the collinearly divergent part given by

$$
\begin{aligned}
V_g^{(R_{q\bar{q}})(C_F,\,\mathrm{coll})}(\vec{q}) &= \frac{2N_F}{(4\pi)^\varepsilon}\int_0^1 dx f_g(x)\int_0^1 d\beta T_R \mathcal{P}_{qg}(\varepsilon,\beta) \\
&\times \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}\vec{k}^2}\Theta(\Lambda^2-\vec{k}^2)S_J^{(2)}(\vec{q};x\beta) \\
&= 2N_F\frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon\int_0^1 dx f_g(x) \\
&\times \int_0^1 d\beta[P_{qg}(\beta)+\varepsilon\beta(1-\beta)]S_J^{(2)}(\vec{q};x\beta)+\mathcal{O}(\varepsilon)\ ,
\end{aligned}
\tag{2.68}
$$

where we have introduced, as before, the cutoff parameter Λ. The finite part is therefore defined by

$$
V_g^{(R_{q\bar{q}})(C_F,\,\mathrm{finite})} = V_g^{(R_{q\bar{q}})(C_F)} - V_g^{(R_{q\bar{q}})(C_F,\,\mathrm{coll})}\ , \tag{2.69}
$$

where one can take $\varepsilon \to 0$ limit and get

$$
\begin{aligned}
V_g^{(R_{q\bar{q}})(C_F,\,\mathrm{finite})} &= 2N_F\int_0^1 dx f_g(x)\int_0^1 d\beta P_{qg}(\beta) \\
&\times \int \frac{d^2\vec{k}}{\pi\vec{k}^2}\Bigg[\frac{\vec{q}^{\,2}}{\vec{k}^2+(\vec{q}-\vec{k})^2}S_J^{(3)}(\vec{k},\vec{q}-\vec{k},x\beta;x) \\
&- \Theta(\Lambda^2-\vec{k}^2)S_J^{(2)}(\vec{q};x\beta)\Bigg]+\mathcal{O}(\varepsilon)\ .
\end{aligned}
\tag{2.70}
$$

The second contribution in (2.64) is

$$
\begin{aligned}
V_g^{(R_{q\bar{q}})(C_A)}(\vec{q}) &= \frac{N_F}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int_0^1 d\beta\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\,2}}{\vec{k}^2+(\vec{q}-\vec{k})^2} \\
&\times T_R\mathcal{P}_{qg}(\varepsilon,\beta)\frac{\beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})}{(\vec{k}-\beta\vec{q})^2}S_J^{(3)}(\vec{q}-\vec{k},\vec{k},x\beta;x)\ .
\end{aligned}
\tag{2.71}
$$

Here the collinear divergence appears for $\vec{k}-\beta\vec{q}=0$ and the integral in this region can be identified with

$$
\begin{aligned}
V_g^{(R_{q\bar{q}})(C_A,\,\mathrm{coll})}(\vec{q}) &= \frac{N_F}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int_0^1 d\beta T_R\mathcal{P}_{qg}(\varepsilon,\beta)\\
&\times\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}(\vec{k}-\beta\vec{q})^2}\Theta(\Lambda^2-(\vec{k}-\beta\vec{q})^2)S_J^{(2)}(\vec{q};x)\\
&= N_F\frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon\left(\frac{1}{3}+\frac{\varepsilon}{6}\right)\\
&\times\int_0^1 dx\frac{C_A}{C_F}f_g(x)S_J^{(2)}(\vec{q};x)+\mathcal{O}(\varepsilon)\;.
\end{aligned}
\tag{2.72}
$$

Then, the finite part can be written as

$$
V_g^{(R_{q\bar{q}})(C_A,\,\mathrm{finite})}=V_g^{(R_{q\bar{q}})(C_A)}-V_g^{(R_{q\bar{q}})(C_A,\,\mathrm{coll})}\;. \tag{2.73}
$$

After the change of variable $\vec{k}\to\vec{q}-\vec{k}$ in (2.71) and (2.72) we have

$$
\begin{aligned}
V_g^{(R_{q\bar{q}})(C_A,\,\mathrm{finite})}(\vec{q}) &= N_F\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int_0^1 d\beta P_{qg}(\beta)\int\frac{d^2\vec{k}}{\pi(\vec{k}-(1-\beta)\vec{q})^2}\\
&\left[\frac{\vec{q}^{\,2}\beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})}{\vec{k}^2(\vec{k}-\vec{q})^2}S_J^{(3)}(\vec{k},\vec{q}-\vec{k},x\beta;x)\right.\\
&\left.-\Theta(\Lambda^2-(\vec{k}-(1-\beta)\vec{q})^2)S_J^{(2)}(\vec{q};x)\right]+\mathcal{O}(\varepsilon)\;.
\end{aligned}
\tag{2.74}
$$

2.4.3 Real corrections: gg intermediate state

The real contribution from the gluon-gluon case is [10, 11, 12, 32]:

$$
\begin{aligned}
V_g^{(R_{gg})}(\vec{q}) &= \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\int_{\beta_0}^{1-\beta_0}d\beta\frac{\vec{q}^{\,2}\mathcal{P}_{gg}(\beta)}{(\vec{k}-\beta\vec{q})^2\vec{k}^2(\vec{k}-\vec{q})^2}\\
&\times\left\{\beta^2(\vec{k}-\vec{q})^2+(1-\beta)^2\vec{k}^2-\beta(1-\beta)\vec{k}\cdot(\vec{q}-\vec{k})\right\}\\
&\times S_J^{(3)}(\vec{q}-\vec{k},\vec{k},x\beta;x)\;.
\end{aligned}
\tag{2.75}
$$

where

$$
\mathcal{P}_{gg}(\beta)=P(\beta)+P(1-\beta),\quad\text{with}\quad P(\beta)=\left(\frac{1}{\beta}+\frac{\beta}{2}\right)(1-\beta)\;.
$$

We note that here the lower integration limit in β is $\beta_0 = \vec{k}^2/s_\Lambda$, whereas the upper limit is $1-\beta$. This comes from the Θ function in the impact factor definition (2.12), which restricts the radiation of either of the two gluons into the central region of rapidity.

Using the symmetry of the integrand under the change of variables describing the two gluons, $\beta \to 1-\beta$ and $\vec{k} \to \vec{q}-\vec{k}$ (thanks to the symmetry property (2.23) of the jet function), we get

$$
\begin{aligned}
V_g^{(R_{gg})}(\vec{q}) = 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^{1-\beta_0} d\beta P(\beta) \\
\frac{\vec{q}^{\,2}}{(\vec{k}-\beta\vec{q})^2\vec{k}^2(\vec{k}-\vec{q})^2}\left\{\beta^2(\vec{k}-\vec{q})^2 + (1-\beta)\vec{k}\cdot(\vec{k}-\beta\vec{q})\right\} \\
\times S_J^{(3)}(\vec{q}-\vec{k},\vec{k},x\beta;x) \equiv V_g^{(R_{gg})(A)}(\vec{q}) + V_g^{(R_{gg})(B)}(\vec{q}) \,.
\end{aligned}
\tag{2.76}
$$

In this form the upper limit of β integration can be put to unity. In $V_g^{(R_{gg})(A)}$ the lower integration limit β_0 can be put equal to zero. Then, after the change of variable $\vec{k} = \beta\vec{l}$, we obtain

$$
\begin{aligned}
V_g^{(R_{gg})(A)}(\vec{q}) &= 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int_0^1 d\beta P(\beta)\beta^{2\varepsilon} \int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}} \\
&\times \frac{\vec{q}^{\,2}}{\vec{l}^{\,2}(\vec{l}-\vec{q})^2} S_J^{(3)}(\vec{q}-\beta\vec{l},\beta\vec{l},x\beta;x) \\
&= 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int_0^1 d\beta P(\beta)\beta^{2\varepsilon} \int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}} \\
&\times \frac{\vec{q}^{\,2}}{\vec{l}^{\,2}+(\vec{l}-\vec{q})^2}\left[\frac{1}{\vec{l}^{\,2}} + \frac{1}{(\vec{l}-\vec{q})^2}\right] S_J^{(3)}(\vec{q}-\beta\vec{l},\beta\vec{l},x\beta;x) \,.
\end{aligned}
\tag{2.77}
$$

In this expression one has both soft and collinear divergences. The soft divergence can be isolated in the counterterm

$$
\begin{aligned}
V_g^{(R_{gg})(A,\,\mathrm{soft})}(\vec{q}) &= 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int_0^1 \frac{d\beta}{\beta^{1-2\varepsilon}} \int \frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{l}^{\,2}+(\vec{l}-\vec{q})^2} \\
&\times \left[\frac{1}{(\vec{l}-\vec{q})^2} + \frac{1}{\vec{l}^{\,2}}\right] S_J^{(2)}(\vec{q};x) \,,
\end{aligned}
\tag{2.78}
$$

which equals

$$
V_g^{(R_{gg})(A,\,\mathrm{soft})}(\vec{q}) = \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\vec{q}^{\,2})^\varepsilon \frac{2C_A}{\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) S_J^{(2)}(\vec{q};x) \,.
\tag{2.79}
$$

After the subtraction of the soft divergence, collinear divergences still appear for $\vec{l}=0$ and $\vec{l}-\vec{q}=0$ and can be isolated by the following two counterterms:

$$V_g^{(R_{gg})(A,\,\mathrm{coll}_1)}(\vec{q}) = 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int\frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}(\vec{q}-\vec{l})^2}\Theta\big(\Lambda^2-(\vec{q}-\vec{l})^2\big) \times \int_0^1 d\beta\beta^{2\varepsilon}\left(P(\beta)-\frac{1}{\beta}\right)S_J^{(2)}(\vec{q};x)\ . \tag{2.80}$$

and

$$V_g^{(R_{gg})(A,\,\mathrm{coll}_2)}(\vec{q}) = 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int\frac{d^{D-2}\vec{l}}{\pi^{1+\varepsilon}\vec{l}^2}\Theta\left(\Lambda^2-\vec{l}^2\right) \times\int_0^1 d\beta\beta^{2\varepsilon}\left(P(\beta)S_J^{(2)}(\vec{q};x(1-\beta))-\frac{1}{\beta}S_J^{(2)}(\vec{q};x)\right)\ . \tag{2.81}$$

These counterterms equal

$$V_g^{(R_{gg})(A,\,\mathrm{coll}_1)}(\vec{q}) = \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon\int_0^1 dx\frac{C_A}{C_F}f_g(x)S_J^{(2)}(\vec{q};x) \times C_A\left(-\frac{11}{6}+\varepsilon\frac{67}{18}\right)+\mathcal{O}(\varepsilon)\ , \tag{2.82}$$

$$V_g^{(R_{gg})(A,\,\mathrm{coll}_2)}(\vec{q}) = \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int_0^1 d\beta(1-\beta)P(1-\beta) \times 2C_A\left[\frac{1}{(1-\beta)_+}+2\varepsilon\left(\frac{\ln(1-\beta)}{1-\beta}\right)_+\right]S_J^{(2)}(\vec{q};x\beta)+\mathcal{O}(\varepsilon)\ , \tag{2.83}$$

where to obtain the last equation we made the change of variable $\beta\to 1-\beta$ and expanded the term $(1-\beta)^{2\varepsilon-1}$. The finite part is therefore defined by

$$V_g^{(R_{gg})(A,\,\mathrm{finite})} = V_g^{(R_{gg})(A)} - V_g^{(R_{gg})(A,\,\mathrm{soft})} - V_g^{(R_{gg})(A,\,\mathrm{coll}_1)} - V_g^{(R_{gg})(A,\,\mathrm{coll}_2)}\ . \tag{2.84}$$

The $V_g^{(R_{gg})(B)}$ term, defined in (2.76), has a collinear divergence for $\vec{k}-\vec{q}=0$. It can be isolated in the following integral:

$$\begin{aligned} V_g^{(R_{gg})(B,\,\mathrm{coll})}(\vec{q}) &= 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int_0^1 d\beta P(\beta) \\ &\times\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}(\vec{k}-\vec{q})^2}\Theta\big(\Lambda^2-(\vec{k}-\vec{q})^2\big)S_J^{(2)}(\vec{q};x\beta) \\ &= \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}(\Lambda^2)^\varepsilon\int_0^1 dx\frac{C_A}{C_F}f_g(x) \\ &\times\int_0^1 d\beta\, 2C_A P(\beta)S_J^{(2)}(\vec{q};x\beta)+\mathcal{O}(\varepsilon)\ , \end{aligned} \tag{2.85}$$

where β_0 has been put equal to zero thanks to the property of lowest order jet function (2.17). Another singularity appears when $\beta \to 0$; in this region we can isolate the term

$$
\begin{aligned}
& V_g^{(R_{gg})(B,\,\mathrm{soft})}(\vec{q}) = \\
& = 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 \frac{d\beta}{\beta} \vec{q}^{\,2} \frac{(1-\beta)\vec{k}\cdot(\vec{k}-\beta\vec{q})}{\vec{k}^2(\vec{q}-\vec{k})^2(\vec{k}-\beta\vec{q})^2} S_J^{(2)}(\vec{q}-\vec{k};x) \\
& = 2\frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \int_{\beta_0}^1 \frac{d\beta}{\beta} \frac{\vec{q}^{\,2}\Theta[(1-\beta)|\vec{k}|-\beta|\vec{q}-\vec{k}|]}{\vec{k}^2(\vec{k}-\vec{q})^2} S_J^{(2)}(\vec{q}-\vec{k};x) \\
& = \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} \ln \frac{s_\Lambda^2}{\vec{k}^2(|\vec{k}|+|\vec{q}-\vec{k}|)^2} S_J^{(2)}(\vec{q}-\vec{k};x) \; .
\end{aligned}
\tag{2.86}
$$

The finite part of $V_g^{(R_{gg})(B)}$ is therefore defined by

$$
V_g^{(R_{gg})(B,\,\mathrm{finite})} = V_g^{(R_{gg})(B)} - V_g^{(R_{gg})(B,\,\mathrm{coll})} - V_g^{(R_{gg})(B,\,\mathrm{soft})} \; . \tag{2.87}
$$

When the gluon part of BFKL counterterm, given in (2.35), is combined with $V_g^{(R_{gg})(B,\,\mathrm{soft})}$ given in (2.86), we see that the dependence on s_Λ disappears and we obtain

$$
\begin{aligned}
& V_g^{(R_{gg})(B,\,\mathrm{soft})}(\vec{q}) + V_g^{(C)}(\vec{q}) = \qquad (2.88) \\
= \; & \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} \ln \frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2} S_J^{(2)}(\vec{q}-\vec{k};x) \; .
\end{aligned}
$$

2.4.4 Final result for the gluon in the initial state

We collect first the contributions which contain singularities given in (2.63), (2.68), (2.72), (2.80), (2.82), (2.83) and (2.85) and get

$$
\begin{aligned}
V_g^{(1)}(\vec{q}) \equiv (V_g^{(V)} + V_g^{(R_{q\bar{q}})(C_F,\,\mathrm{coll})} + V_g^{(R_{q\bar{q}})(C_A,\,\mathrm{coll})} V_g^{(R_{gg})(A,\,\mathrm{soft})} \\
+ V_g^{(R_{gg})(A,\,\mathrm{coll}_1)} + V_g^{(R_{gg})(A,\,\mathrm{coll}_2)} + V_g^{(R_{gg})(B\,\mathrm{coll})})(\vec{q}) \\
= \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}\int_0^1 dx \frac{C_A}{C_F} f_g(x) \Bigg\{ \Bigg[(\vec{q}^{\,2})^\varepsilon \left(\frac{11C_A}{6} - \frac{N_F}{3} - C_A \ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)\right) \\
-2(\Lambda^2)^\varepsilon \left(\frac{11C_A}{6} - \frac{N_F}{3}\right) + \varepsilon\left(\frac{\pi^2 C_A}{2} + \frac{13N_F}{18}\right)\Bigg] S_J^{(2)}(\vec{q};x) \\
+(\Lambda^2)^\varepsilon \int_0^1 d\beta \left(P_{gg}(\beta) + 2N_F \frac{C_F}{C_A} P_{qg}(\beta)\right) S_J^{(2)}(\vec{q};x) \\
+2\varepsilon \int_0^1 d\beta \Bigg[N_F \frac{C_F}{C_A}(1-\beta)\beta \\
+2C_A \left(\frac{\ln(1-\beta)}{1-\beta}\right)_+ (1-\beta)P(1-\beta)\Bigg] S_J^{(2)}(\vec{q};x) \Bigg\} + \mathcal{O}(\varepsilon) \, .
\end{aligned} \tag{2.89}
$$

Then, we collect the contributions given in (2.70), (2.74), (2.84), (2.87), and get (transforming to the form used in [8])

$$
\begin{aligned}
V_g^{(2)}(\vec q) &\equiv (V_g^{(R_{q\bar q})(C_F,\,\text{finite})}+V_g^{(R_{q\bar q})(C_A,\,\text{finite})}+V_g^{(R_{gg})(C_A,\,\text{finite})}+V_g^{(R_{gg})(B,\,\text{finite})})(\vec q)\\
&= \int_0^1 dx f_g(x)\int_0^1 d\beta\Bigg[2N_F P_{qg}(\beta)\int\frac{d^2\vec k}{\pi\vec k^2}\Bigg\{\frac{\vec q^{\,2}}{\vec k^2+(\vec q-\vec k)^2}\\
&\qquad\times S_J^{(3)}(\vec k,\vec q-\vec k,x\beta;x)-\Theta(\Lambda^2-\vec k^2)S_J^{(2)}(\vec q;x\beta)\Bigg\}\\
&\quad+N_F\frac{C_A}{C_F}P_{qg}(\beta)\int\frac{d^2\vec k}{\pi(\vec k-(1-\beta)\vec q)^2}\Bigg\{\frac{\vec q^{\,2}\beta(1-\beta)\vec k\cdot(\vec q-\vec k)}{\vec k^2(\vec k-\vec q)^2}\\
&\quad\times S_J^{(3)}(\vec k,\vec q-\vec k,x\beta;x)-\Theta(\Lambda^2-(\vec k-(1-\beta)\vec q)^2)S_J^{(2)}(\vec q;x)\Bigg\}\Bigg]\\
&\quad+\int_0^1 dx\,2C_A\frac{C_A}{C_F}f_g(x)\Bigg[\int_0^1\frac{d\beta}{(1-\beta)_+}[(1-\beta)P(1-\beta)]\int\frac{d^2\vec l}{\pi\vec l^{\,2}}\\
&\quad\times\Bigg\{\frac{\vec q^{\,2}}{\vec l^{\,2}+(\vec l-\vec q)^2}\Bigg(S_J^{(3)}(\vec q-(1-\beta)\vec l,(1-\beta)\vec l,x(1-\beta);x)\\
&\qquad+S_J^{(3)}(\beta\vec q+(1-\beta)\vec l),(1-\beta)(\vec q-\vec l),x(1-\beta);x)\Bigg) \qquad (2.90)\\
&\qquad-\Theta\left(\Lambda^2-\vec l^{\,2}\right)\left(S_J^{(2)}(\vec q;x\beta)+S_J^{(2)}(\vec q;x)\right)\Bigg\}\\
&\quad+\int_0^1 d\beta\int\frac{d^2\vec k}{\pi}\Bigg\{P(\beta)\Bigg(\frac{\vec q^{\,2}(1-\beta)\vec k\cdot(\vec k-\beta\vec q)}{(\vec k-\beta\vec q)^2(\vec k-\vec q)^2\vec k^2}S_J^{(3)}(\vec q-\vec k,\vec k,x\beta;x)\\
&\qquad-\frac{1}{(\vec k-\vec q)^2}\Theta\left(\Lambda^2-(\vec k-\vec q)^2\right)S_J^{(2)}(\vec q;x\beta)\Bigg)\\
&\qquad-\frac{1}{\beta}\frac{\vec q^{\,2}\Theta[(1-\beta)|\vec q-\vec k|-\beta|\vec k|]}{\vec k^2(\vec q-\vec k)^2}S_J^{(2)}(\vec k;x)\Bigg\}\Bigg]\,.
\end{aligned}
$$

Besides, we define

$$
\begin{aligned}
V_g^{(3)}(\vec q) &\equiv (V_g^{(R_{gg})(B,\,\text{soft})}+V_g^{(C)})(\vec q) \qquad (2.91)\\
&= \frac{C_A}{(4\pi)^\varepsilon}\int_0^1 dx\frac{C_A}{C_F}f_g(x)\int\frac{d^{D-2}\vec k}{\pi^{1+\varepsilon}}\frac{\vec q^{\,2}}{\vec k^2(\vec k-\vec q)^2}\ln\frac{s_0}{(|\vec k|+|\vec q-\vec k|)^2}S_J^{(2)}(\vec q-\vec k;x)\,,
\end{aligned}
$$

given in Eq. (2.89).

Another contribution originates from the collinear and charge renormalization counterterms, see Eqs. (2.32) and (2.34),

$$V_g^{(4)}(\vec{q}) = \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\int_0^1 dx f_g(x)\Bigg[(\mu_R^2)^\varepsilon\left(\frac{11C_A}{6}-\frac{N_F}{3}\right)\frac{C_A}{C_F}S_J^{(2)}(\vec{q};x) - (\mu_F^2)^\varepsilon\int_0^1 d\beta\left[2N_F P_{qg}(\beta)+\frac{C_A}{C_F}P_{gg}(\beta)\right]S_J^{(2)}(\vec{q};x\beta)\Bigg]. \tag{2.92}$$

Finally, the gluon part of the jet impact factor is given by the sum of the above four contributions and can be presented as a sum of two terms:

$$V_g^{(I)}(\vec{q}) = \int_0^1 dx\frac{C_A}{C_F}f_g(x)\Bigg[\frac{C_A}{(4\pi)^\varepsilon}\int\frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}}\frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2}\ln\frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2}S_J^{(2)}(\vec{k};x) - C_A\ln\left(\frac{s_0}{\vec{q}^{\,2}}\right)(\vec{q}^{\,2})^\varepsilon\frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon}\frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)}S_J^{(2)}(\vec{q};x)\Bigg] \tag{2.93}$$

and

$$V_g^{(II)}(\vec{q}) = V_g^{(2)}(\vec{q}) + \int_0^1 dx\frac{C_A}{C_F}f_g(x) \times\Bigg\{\left[\left(\frac{11C_A}{6}-\frac{N_F}{3}\right)\ln\frac{\vec{q}^{\,2}\mu_R^2}{\Lambda^4}+C_A\frac{\pi^2}{2}+\frac{13}{18}N_F\right]S_J^{(2)}(\vec{q};x) + \int_0^1 d\beta\left[P_{gg}(\beta)+2N_F\frac{C_F}{C_A}P_{qg}(\beta)\right]\ln\frac{\Lambda^2}{\mu_F^2}S_J^{(2)}(\vec{q};x\beta) + \int_0^1 d\beta\left[4\left(\frac{ln(1-\beta)}{1-\beta}\right)_+[(1-\beta)P(1-\beta)]C_A + 2N_F\frac{C_F}{C_A}\beta(1-\beta)\right]S_J^{(2)}(\vec{q};x\beta)\Bigg\}. \tag{2.94}$$

2.4.5 Infrared finiteness of the jet impact factor

The NLO correction to the jet vertex (impact factor) has the form

$$\frac{d\Phi_J^{(1)}(\vec{q})}{dJ} = \frac{\alpha_s}{2\pi}\Phi_q^{(0)}V(\vec{q}), \qquad V(\vec{q}) = V^{(I)}(\vec{q}) + V^{(II)}(\vec{q}) \tag{2.95}$$

where each part is the sum of the quark and gluon contributions,

$$V^{(I)}(\vec{q}) = V_q^{(I)}(\vec{q}) + V_g^{(I)}(\vec{q}), \qquad V^{(II)}(\vec{q}) = V_q^{(II)}(\vec{q}) + V_g^{(II)}(\vec{q})$$

given in Eqs. (2.60), (2.3.3) and in Eqs. (2.93), (2.94), respectively. $V_q^{(II)}(\vec{q})$ and $V_g^{(II)}(\vec{q})$ are manifestly finite. For $V^{(I)}(\vec{q})$ we have

$$
\begin{aligned}
V_q^{(I)}(\vec{q}) = & \int_0^1 dx \left(\sum_{a=q,\bar{q}} f_a(x) + \frac{C_A}{C_F} f_g(x) \right) \\
& \times \left[\frac{C_A}{(4\pi)^\varepsilon} \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} \ln \frac{s_0}{(|\vec{k}|+|\vec{q}-\vec{k}|)^2} S_J^{(2)}(\vec{k};x) \right. \\
& \left. - C_A \ln\left(\frac{s_0}{\vec{q}^{\,2}}\right) (\vec{q}^{\,2})^\varepsilon \frac{\Gamma[1-\varepsilon]}{\varepsilon(4\pi)^\varepsilon} \frac{\Gamma^2(1+\varepsilon)}{\Gamma(1+2\varepsilon)} S_J^{(2)}(\vec{q};x) \right] .
\end{aligned}
\tag{2.96}
$$

Having the explicit form of the lowest order jet function (2.17), it is easy to see that the integration of $V^{(I)}(\vec{q})$ over $\vec{q}$ with any function, regular at $\vec{q} = \vec{k}_J$, will give some finite result. In particular, the finite result will be obtained after the convolution of $V^{(I)}(\vec{q})$ with BFKL Green's function, see Eq. (2.16), which is required for the calculation of the jet cross section.

The divergence in (2.96) arises from virtual corrections and, precisely, from the factor $(s_0/\vec{q}^{\,2})^{\omega(\vec{q}^{\,2})}$ entering the definition of the impact factor. In the computation of physical impact factors this divergence is cancelled by the one arising from the integration in the first term of Eq. (2.96), which is related with real emission. In the calculation of the jet vertex the $\vec{q}$ integration is "opened" and, therefore, there is no way to get the divergence needed to balance the one arising from virtual corrections. However, in the construction of any physical cross section, the jet vertex is to be convoluted with the BFKL Green's function, which implies the integration over the Reggeon transverse momentum $\vec{q}$ and it is after this integration that the divergence cancels.

2.4.6 Energy scale

In our approach the energy scale s_0 remains untouched and need not be fixed at any definite scale. The dependence on s_0 will disappear in the next-to-leading logarithmic approximation in any physical cross section in which jet vertices are used. However, the dependence on this energy scale will survive in terms beyond this approximation and will provide a parameter to be optimized with the method adopted in Refs. [23, 33].

In order to compare our results with the ones of [7, 8], we need to perform the transition from the standard BFKL scheme with arbitrary energy scale s_0 to the one used in [7, 8], where the scale of energy depends on the Reggeon momentum. The change to the scheme where the energy scale s_0 is replaced to any factorizable scale $\sqrt{f_1(\vec{q}_1^{\,2})f_2(\vec{q}_2^{\,2})}$ leads to the following modification of each impact factor ($i = 1, 2$), see [34],

$$\Phi_i(\vec{q}; f_i(\vec{q}^{\,2})) = \Phi_i(\vec{q}; s_0) + \frac{1}{2}\int d^{D-2}\vec{k}\, \Phi_i^{(0)}(\vec{k}) \ln\left(\frac{f_i(\vec{k}^2)}{s_0}\right) K^{(0)}(\vec{k}, \vec{q})\, \frac{\vec{q}^{\,2}}{\vec{k}^2}\,, \tag{2.97}$$

where $\Phi_i^{(0)}$ and $K^{(0)}$ are the lowest order impact factor and BFKL kernel. Therefore changing from s_0 to $\sqrt{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$ we obtain the following replacement in our result for the jet impact

$$\begin{aligned} V^{(I)}(\vec{q}) \to \bar{V}^{(I)}(\vec{q}) &= \int_0^1 dx \left(\sum_{a=q,\bar{q}} f_a(x) + \frac{C_A}{C_F} f_g(x)\right) \\ &\times \left[\frac{C_A}{(4\pi)^{\varepsilon}} \int \frac{d^{D-2}\vec{k}}{\pi^{1+\varepsilon}} \frac{\vec{q}^{\,2}}{\vec{k}^2(\vec{k}-\vec{q})^2} \ln \frac{\vec{k}^2}{(|\vec{k}| + |\vec{q}-\vec{k}|)^2} S_J^{(2)}(\vec{k}; x)\right]. \end{aligned} \tag{2.98}$$

Note that $\bar{V}^{(I)}(\vec{q})$ is not singular at $\vec{q} \to \vec{q}$ and, therefore, it can be calculated at $D = 4$. Such contribution to the jet impact factors, $\bar{V}^{(I)}(\vec{q})$, in the considered scheme with $s_0 = \sqrt{\vec{q}_1^{\,2}\vec{q}_2^{\,2}}$ produces a completely equivalent effect on the physical jet cross section as the factors H_L and H_R which enter Eq. (76) of [8] (see Eqs. (101), (102) in [7] for the definition of H_L, H_R).

Therefore, for the final comparison one needs to consider our results for $V_q^{(II)}(\vec{q})$ and $V_g^{(II)}(\vec{q})$ (modulo the appropriate normalization factor) with the ones given in Eq. (105) of [7] and Eq. (67) of [8] for the quark and gluon contributions, respectively. For this purpose we identify, following [7, 8], the Λ parameter with the collinear factorization scale μ_F. In the gluon contribution we found a complete agreement.

2.5 Summary

Eventually we have recalculated the jet vertices for the cases of quark and gluon in the initial state, first found in the papers by Bartels et al. [7, 8]. Our approach is more straightforward and more general, since the starting point of our calculation is the known general expression for NLO BFKL impact factors, given in Ref. [25], applied to the special case of partons in the initial state.

Chapter 3

Further Developments

3.1 Small-cone approximation

Another approach to obtain the NLO impact factor for the production of forward jets is the small-cone approximation (SCA) [37, 38] and is developed in [39]. In this paper the authors starting from the totally inclusive NLO parton impact factors calculated in [10, 11] used SCA to obtain an expression for small jet cone aperture in the rapidity azimuthal angle plane.

At LO the totally inclusive parton impact factor takes contribution from a one-particle intermediate state; this leads to fix the kinematics of the produced parton by the jet kinematics.

So inserting into the inclusive impact factor the delta function which depends on the jet variables transverse momentum $\vec{k}$ and longitudinal fraction α, they get

$$\frac{d\Phi^J}{\vec{q}^{\,2}} = \mathcal{C} \int d\alpha \frac{d^2\vec{k}}{\vec{k}^2} dx \delta^{(2)}\left(\vec{k}-\vec{q}\right) \delta(\alpha - x) \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right) , \quad (3.1)$$

with

$$d\Phi = \mathcal{C}\, dx \left(\frac{C_A}{C_F} f_g(x) + \sum_{a=q,\bar{q}} f_a(x)\right), \quad \mathcal{C} = 2\pi\alpha_s\sqrt{\frac{2C_F}{C_A}} \, . \quad (3.2)$$

At NLO there are both the virtual correction and also two-particle production in the parton-Reggeon collision.

For the two particle production the jet can be either produced by one of the two partons or by both together. If the produced parton are a and b there can be the following contributions:

- Only the parton a generates the jet, while the parton b can have arbitrary kinematics, provided that it lies outside the jet cone;
- the same but $a \leftrightarrow b$;
- the two parton a and b *both generate the jet.*

Introducing the relative rapidity and azimuthal angle between the two partons:

$$\Delta y = \frac{1}{2} \ln \frac{\zeta^2(\vec{k}-\vec{q})^2}{\bar{\zeta}^2 \vec{k}^2}, \quad \Delta\phi = \arccos \frac{\vec{q}\cdot\vec{k}-\vec{k}^2}{|\vec{k}||\vec{q}-\vec{k}|}, \quad \bar{\zeta} \equiv 1-\zeta. \tag{3.3}$$

If the parton with momentum $\vec{k}$ and longitudinal fraction ζ generate the jet, and if

$$\vec{\Delta} = \vec{q} - \frac{\vec{k}}{\zeta} \tag{3.4}$$

the condition of cone aperture smaller than R in the rapidity-azimuthal angle plane becomes

$$\Delta\phi^2 + \Delta y^2 \leq R^2 \tag{3.5}$$

and then

$$|\vec{\Delta}| \leq \frac{\bar{\zeta}}{\zeta}|\vec{k}|R. \tag{3.6}$$

If the jet is formed by both partons the jet momentum and jet longitudinal fraction:

$$\vec{k} = \vec{k}_1 + \vec{k}_2; \quad 1 = \zeta + \bar{\zeta}\,. \tag{3.7}$$

The relative rapidity and azimuthal angle between the jet and the first (second) parton are:

$$\Delta y_1 = \frac{1}{2}\ln\frac{\vec{k}_1^2}{\zeta^2\vec{k}^2}, \quad \Delta\phi_1 = \arccos\frac{\vec{k}\cdot\vec{k}_1}{|\vec{k}_1||\vec{k}|}\,, \tag{3.8}$$

$$\Delta y_2 = \frac{1}{2}\ln\frac{(\vec{k}_1-\vec{k})^2}{\bar{\zeta}^2\vec{k}^2}, \quad \Delta\phi_2 = \arccos\frac{\vec{k}\cdot(\vec{k}-\vec{k}_1)}{|\vec{k}||\vec{k}-\vec{k}_1|}\,. \tag{3.9}$$

Introducing now the vector $\vec{\Delta}$ as

$$\vec{k}_1 = \zeta \vec{k} + \vec{\Delta} \tag{3.10}$$

the requirement that both partons are inside the cone:

$$|\vec{\Delta}| \leq R |\vec{k}| \min(\zeta, \bar{\zeta}) \,. \tag{3.11}$$

The result of the paper is that working in

$$D = 4 + 2\varepsilon \tag{3.12}$$

dimensions and calculating the NLO impact factor in the (ν, n)-representation, the authors found the following results for an incoming quark:

$$\begin{aligned}
I_q = & \frac{\alpha_s}{2\pi} \left(\vec{k}^2\right)^{\gamma - \frac{n}{2}} \left(\vec{k} \cdot \vec{l}\right)^n \int_\alpha^1 \frac{d\zeta}{\zeta} \sum_{a=q,\bar{q}} f_a \left(\frac{\alpha}{\zeta}\right) \\
& \times \left[\left\{ P_{qq}(\zeta) + \frac{C_A}{C_F} P_{gq}(\zeta) \right\} \ln \frac{\vec{k}^2}{\mu_F^2} - 2\zeta^{-2\gamma} \ln R \{ P_{qq}(\zeta) + P_{gq}(\zeta) \} \right. \\
& - \frac{\beta_0}{2} \ln \frac{\vec{k}^2}{\mu_R^2} \delta(1-\zeta) + C_A \delta(1-\zeta) \left\{ \chi(n,\gamma) \ln \frac{s_0}{\vec{k}^2} + \frac{85}{18} + \frac{\pi^2}{2} \right. \\
& \left. + \frac{1}{2} \left(\psi'\left(1 + \gamma + \frac{n}{2}\right) - \psi'\left(\frac{n}{2} - \gamma\right) - \chi^2(n,\gamma) \right) \right\} \\
& + (1+\zeta^2) \left\{ C_A \left(\frac{(1+\zeta^{-2\gamma})\chi(n,\gamma)}{2(1-\zeta)_+} - \zeta^{-2\gamma} \left(\frac{\ln(1-\zeta)}{1-\zeta} \right)_+ \right) \right. \\
& \left. + \left(C_F - \frac{C_A}{2} \right) \left[\frac{\bar{\zeta}}{\zeta^2} I_2 - \frac{2\ln\zeta}{1-\zeta} + 2 \left(\frac{\ln(1-\zeta)}{1-\zeta} \right)_+ \right] \right\} \\
& + \delta(1-\zeta) \left(C_F \left(3\ln 2 - \frac{\pi^2}{3} - \frac{9}{2} \right) - \frac{5n_f}{9} \right) + C_A \zeta \\
& \left. + C_F \bar{\zeta} + \frac{1+\bar{\zeta}^2}{\zeta} \left\{ C_A \frac{\bar{\zeta}}{\zeta} I_1 + 2C_A \ln \frac{\bar{\zeta}}{\zeta} + C_F \zeta^{-2\gamma} (\chi(n,\gamma) - 2\ln\bar{\zeta}) \right\} \right] .
\end{aligned} \tag{3.13}$$

and for an incoming gluon:

$$
\begin{aligned}
I_g &= \frac{\alpha_s}{2\pi}\left(\vec{k}^2\right)^{\gamma-\frac{n}{2}}\left(\vec{k}\cdot\vec{l}\right)^n \int_\alpha^1 \frac{d\zeta}{\zeta} f_g\left(\frac{\alpha}{\zeta}\right)\frac{C_A}{C_F} \\
&\times \left\{\left\{P_{gg}(\zeta)+2n_f\frac{C_F}{C_A}P_{qg}(\zeta)\right\}\ln\frac{\vec{k}^2}{\mu_F^2}-2\zeta^{-2\gamma}\ln R\{P_{gg}(\zeta)+2n_fP_{qg}(\zeta)\}\right. \\
&- \frac{\beta_0}{2}\ln\frac{\vec{k}^2}{4\mu_R^2}\delta(1-\zeta)+C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^2}+\frac{1}{12}+\frac{\pi^2}{6}\right. \\
&+ \left.\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\} \\
&+ 2C_A(1-\zeta^{-2\gamma})\left(\left(\frac{1}{\zeta}-2+\zeta\bar{\zeta}\right)\ln\bar{\zeta}+\frac{\ln(1-\zeta)}{1-\zeta}\right) \\
&+ C_A\left[\frac{1}{\zeta}+\frac{1}{(1-\zeta)_+}-2+\zeta\bar{\zeta}\right]\left((1+\zeta^{-2\gamma})\chi(n,\gamma)-2\ln\zeta+\frac{\bar{\zeta}^2}{\zeta^2}I_2\right) \\
&+ \left. n_f\left[2\zeta\bar{\zeta}\frac{C_F}{C_A}+(\zeta^2+\bar{\zeta}^2)\left(\frac{C_F}{C_A}\chi(n,\gamma)+\frac{\bar{\zeta}}{\zeta}I_3\right)-\frac{1}{12}\delta(1-\zeta)\right]\right\} .
\end{aligned}
\tag{3.14}
$$

The final result for the NLO jet vertex is given by

$$
\frac{d\Phi^J(\nu,n)}{d\alpha\, d^{2+2\varepsilon}\vec{k}} \equiv \frac{IC}{\pi\sqrt{2}\vec{k}^2} \tag{3.15}
$$

where

$$
I = I_q + I_g\ . \tag{3.16}
$$

In these results soft and virtual infrared divergences cancel each other while infrared collinear divergences are compensated by the PDFs' renormalization counter terms. The remaining ultraviolet divergences instead by the renormalization of the QCD coupling.

In this approach the energy scale s_0 is an arbitrary parameter, that need not be fixed at any definite scale but the dependence on s_0 will disappear in the next-to-leading logarithmic approximation in any physical cross-section.

3.2 Mueller-Navelet jet process cross section

The small-cone approximation is used in Ref. [40] to obtain prediction for the Mueller-Navelet jet process cross section and for the azimuthal angle decorrelation observables in the SCA. According to BFKL approach the cross section reads:

$$\begin{aligned}\frac{d\sigma}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}} &= \sum_{i,j=q,\bar{q},g}\int_0^1 dx_1 \int_0^1 dx_2 \\ &\times f_i(x_1,\mu_F)f_i(x_2,\mu_F)\frac{d\hat{\sigma}_{i,j}(x_1x_2s,\mu_F)}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}}\ ,\end{aligned} \tag{3.17}$$

as already discussed in the previous chapter. Here i,j are the parton types, $f_i(x,\mu_F)$ is the initial proton PDFs, the longitudinal fractions of the partons involved in the hard subprocess are $x_{1,2}$, μ_F is the factorization scale, $d\hat{\sigma}_{i,j}(x_1x_2s,\mu_F)$ is the partonic cross section for the production of jets and $\hat{s}=x_1x_2s$ is the squared center-of-mass energy of the parton-parton collision subprocess and the authors use the $\overline{\text{MS}}$ scheme for the ultraviolet and collinear factorization. The cross section for the hard subprocess is:

$$\begin{aligned}\frac{d\hat{\sigma}_{i,j}(x_1x_2s)}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}} &= \frac{1}{(2\pi)^2}\int\frac{d^2q_1}{\vec{q}_1^{\,2}}V_i(\vec{q}_1,s_0,x_1;\vec{k}_{J_1},x_{J_1}) \\ \times\int\frac{d^2q_2}{\vec{q}_2^{\,2}}V_j(-\vec{q}_2,s_0,x_2;\vec{k}_{J_2},x_{J_2}) &\times\int_{\delta-i\infty}^{\delta+i\infty}\frac{d\omega}{2\pi i}\left(\frac{x_1x_2s}{s_0}\right)^{\omega}G_\omega(\vec{q}_1,\vec{q}_2)\ .\end{aligned} \tag{3.18}$$

In NLA accuracy $V_i(\vec{q}_1,s_0,x_1;\vec{k}_{J_1},x_{J_1})$ and $V_j(-\vec{q}_2,s_0,x_2;\vec{k}_{J_2},x_{J_2})$ are the jet vertices (impact factors) describing the transitions parton $i(x_1p_1)\to$ jet(k_{J_1}) and parton $j(x_2p_2)\to$ jet(k_{J_2}), in the scattering off a Reggeized gluon with transverse momentum $\vec{q}_1$ and $\vec{q}_2$, respectively.

The authors work in the transverse momentum representation, defined by

$$\hat{\vec{q}}|\vec{q}_i\rangle = \vec{q}_i|\vec{q}_i\rangle, \tag{3.19}$$

$$\langle\vec{q}_1|\vec{q}_2\rangle = \delta^{(2)}(\vec{q}_1-\vec{q}_2), \qquad \langle A|B\rangle = \langle A|\vec{k}\rangle\langle\vec{k}|B\rangle = \int d^2k A(\vec{k})B(\vec{k})\ ; \tag{3.20}$$

the kernel of the operator $\hat{K}$ is

$$K(\vec{q}_2,\vec{q}_1) = \langle\vec{q}_2|\hat{K}|\vec{q}_1\rangle \tag{3.21}$$

The kernel is given as an expansion in the strong coupling,

$$\hat{K} = \bar{\alpha}_s \hat{K}^0 + \bar{\alpha}_s^2 \hat{K}^1 \ , \tag{3.22}$$

where

$$\bar{\alpha}_s = \frac{\alpha_s N_c}{\pi} \tag{3.23}$$

and N_c is the number of colors. In Eq. (3.22) $\hat{K}^0$ is the BFKL kernel in the LLA, $\hat{K}^1$ represents the NLA correction. With NLA accuracy this solution is

$$\hat{G}_\omega = (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} \left(\bar{\alpha}_s^2 \hat{K}^1\right) (\omega - \bar{\alpha}_s \hat{K}^0)^{-1} + \mathcal{O}\left[\left(\bar{\alpha}_s^2 \hat{K}^1\right)^2\right] . \tag{3.24}$$

The basis of eigenfunctions of the LLA kernel,

$$\hat{K}^0 |n,\nu\rangle = \chi(n,\nu)|n,\nu\rangle, \quad \chi(n,\nu) = 2\psi(1) - \psi\left(\frac{n}{2} + \frac{1}{2} + i\nu\right) - \psi\left(\frac{n}{2} + \frac{1}{2} - i\nu\right) , \tag{3.25}$$

is given by the following set of functions:

$$\langle \vec{q}\,|n,\nu\rangle = \frac{1}{\pi\sqrt{2}} (\vec{q}^{\,2})^{i\nu - \frac{1}{2}} e^{in\phi} , \tag{3.26}$$

here ϕ is the azimuthal angle of the vector $\vec{q}$ counted from some fixed direction in the transverse space, $\cos\phi \equiv q_x/|\vec{q}\,|$. The action of the full NLA BFKL kernel on these functions may be expressed as follows:

$$\hat{K}|n,\nu\rangle = \bar{\alpha}_s(\mu_R)\chi(n,\nu)|n,\nu\rangle + \bar{\alpha}_s^2(\mu_R)\left(\chi^{(1)}(n,\nu) + \frac{\beta_0}{4N_c}\chi(n,\nu)\ln(\mu_R^2)\right)|n,\nu\rangle$$
$$+\bar{\alpha}_s^2(\mu_R)\frac{\beta_0}{4N_c}\chi(n,\nu)\left(i\frac{\partial}{\partial\nu}\right)|n,\nu\rangle \ , \tag{3.27}$$

where μ_R is the renormalization scale of the QCD coupling, the first term represents the action of LLA kernel, while the second and the third ones stand for the diagonal and the non-diagonal parts of the NLA kernel and

$$\beta_0 = \frac{11N_c}{3} - \frac{2n_f}{3}, \tag{3.28}$$

where n_f is the number of active quark flavors.
The function $\chi^{(1)}(n,\nu)$

$$\chi^{(1)} = -\frac{\beta_0}{8N_c}\left(\chi^2(n,\nu) - \frac{10}{3}\chi(n,\nu) - i\chi'(n,\nu)\right) + \bar{\chi}(n,\nu) \ , \tag{3.29}$$

where

$$\bar{\chi}(n,\nu) = -\frac{1}{4}\Bigg[\frac{\pi^2-4}{3}\chi(n,\nu) - 6\zeta(3) - \chi''(n,\nu) + 2\phi(n,\nu) + 2\phi(n,-\nu)$$
$$+\frac{\pi^2 \sinh(\pi\nu)}{2\nu\cosh^2(\pi\nu)}\left(\left(3+\left(1+\frac{n_f}{N_c^3}\right)\frac{11+12\nu^2}{16(1+\nu^2)}\right)\delta_{n0}\right.$$
$$\left.\left. -\left(1+\frac{n_f}{N_c^3}\right)\frac{1+4\nu^2}{32(1+\nu^2)}\delta_{n2}\right)\right] \tag{3.30}$$

$$\phi(n,\nu) = \sum_{k=0}^{\infty}\frac{(-1)^{k+1}}{k+(n+1)/2+i\nu}\Bigg[\psi'(k+n+1)-\psi'(k+1)$$
$$+(-1)^{k+1}(\beta'(k+n+1)+\beta'(k+1))$$
$$-\frac{1}{k+(n+1)/2+i\nu}(\psi(k+n+1)-\psi(k+1))\Bigg], \tag{3.31}$$
$$\beta'(z) = \frac{1}{4}\left[\psi'\left(\frac{z+1}{2}\right)-\psi'\left(\frac{z}{2}\right)\right], \quad \mathrm{Li}_2(x) = -\int_0^x dt\frac{\ln(1-t)}{t}\ .$$

Here and below $\chi'(n,\nu) = d\chi(n,\nu)/d\nu$ and $\chi''(n,\nu) = d^2\chi(n,\nu)/d^2\nu$.

For the quark and the gluon jet vertices in (3.2) the projection onto the eigenfunctions of LO BFKL kernel, *i.e.* the transfer to the (ν,n)-representation, is done as follows:

$$\frac{V(\vec{q}_1)}{\vec{q}_1^{\ 2}} = \sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\nu\Phi_1(\nu,n)\langle n,\nu|\vec{q}_1\rangle,$$
$$\frac{V(-\vec{q}_2)}{\vec{q}_2^{\,2}} = \sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty} d\nu\Phi_2(\nu,n)\langle\vec{q}_2|n,\nu\rangle,$$
$$\Phi_1(\nu,n) = \int d^2q_1\frac{V(\vec{q}_1)}{\vec{q}_1^{\,2}}\frac{1}{\pi\sqrt{2}}(\vec{q}_1^{\,2})^{i\nu-\frac{1}{2}}e^{in\phi_1},$$
$$\Phi_2(\nu,n) = \int d^2q_2\frac{V(-\vec{q}_2)}{\vec{q}_2^{\,2}}\frac{1}{\pi\sqrt{2}}(\vec{q}_2^{\,2})^{-i\nu-\frac{1}{2}}e^{-in\phi_2}\ . \tag{3.32}$$

The vertices can be represented as an expansion in α_s,

$$\Phi_{1,2}(n,\nu) = \alpha_s(\mu_R)v_{1,2}(n,\nu) + \alpha_s^2(\mu_R)v_{1,2}^{(1)}(n,\nu)\ . \tag{3.33}$$

The explicit forms of LLA and NLA jet vertices in the (ν,n)-representation both for the quark and gluon cases can be found in (3.23). In particular for the

LLA quark vertices one has

$$
\begin{aligned}
v_1^q(n,\nu) &= 2\sqrt{\frac{C_F}{C_A}}(\vec{k}_{J_1}^2)^{i\nu-\frac{3}{2}}e^{in\phi_{J_1}}\delta(x_{J_1}-x_1), \\
v_2^q(n,\nu) &= 2\sqrt{\frac{C_F}{C_A}}(\vec{k}_{J_2}^2)^{-i\nu-\frac{3}{2}}e^{-in(\phi_{J_2}+\pi)}\delta(x_{J_2}-x_2)\ ,
\end{aligned}
\tag{3.34}
$$

where $C_A = N_c, C_F = (N_c-1)/2N_c$. The partonic cross section can be written with NLA accuracy as follows

$$
\begin{aligned}
&\frac{d\hat{\sigma}(x_1x_2s)}{dx_{J_1}dx_{J_2}d^2k_{J_1}d^2k_{J_2}} = \frac{1}{(2\pi)^2}\sum_{n=-\infty}^{+\infty}\int_{-\infty}^{+\infty}d\nu\left(\frac{x_1x_2s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu)} \\
&\times \alpha_s^2(\mu_R)v_1(n,\nu)v_2(n,\nu)\left[1+\alpha_s(\mu_R)\left(\frac{v_1^{(1)}(n,\nu)}{v_1(n,\nu)}+\frac{v_2^{(1)}(n,\nu)}{v_2(n,\nu)}\right)\right. \\
&+\bar{\alpha}_s^2(\mu_R)\ln\left(\frac{x_1x_2s}{s_0}\right)\left(\bar{\chi}(n,\nu)+\frac{\beta_0}{8N_c}\chi(n,\nu)\left[-\chi(n,\nu)+\frac{10}{3}\right.\right. \\
&\left.\left.\left.+i\frac{d\ln\left(\frac{v_1(n,\nu)}{v_2(n,\nu)}\right)}{d\nu}+2\ln\mu_R^2\right]\right)\right]\ .
\end{aligned}
\tag{3.35}
$$

The differential cross section has the form

$$
\frac{d\sigma}{dy_{J_1}dy_{J_2}d|\vec{k}_{J_1}|d|\vec{k}_{J_2}|d\phi_{J_1}d\phi_{J_2}} = \frac{1}{(2\pi)^2}\left[\mathcal{C}_0+\sum_{n=1}^{\infty}2\cos(n\phi)\mathcal{C}_n\right]\ , \tag{3.36}
$$

where $\phi = \phi_{J_1}-\phi_{J_2}-\pi$, and

$$
\mathcal{C}_m = \int_0^{2\pi}d\phi_{J_1}\int_0^{2\pi}d\phi_{J_2}\cos[m(\phi_{J_1}-\phi_{J_2}-\pi)]\frac{d\sigma}{dy_{J_1}dy_{J_2}d|\vec{k}_{J_1}|d|\vec{k}_{J_2}|d\phi_{J_1}d\phi_{J_2}}\ . \tag{3.37}
$$

In particular, taking into account the Jacobian of the transformation from the variables $\vec{k}_{J_i}$, x_{J_i} to the variables $|\vec{k}_{J_i}|, y_{J_i}$, and the ν-dependence of LLA jet vertices, see (3.34), we get

$$\mathcal{C}_n = \frac{x_{J_1} x_{J_2}}{|\vec{k}_{J_1}||\vec{k}_{J_2}|} \int_{-\infty}^{+\infty} d\nu \left(\frac{x_{J_1} x_{J_2} s}{s_0}\right)^{\bar{\alpha}_s(\mu_R)\chi(n,\nu)} \tag{3.38}$$
$$\times \alpha_s^2(\mu_R) c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1}) c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})$$
$$\times \left[1 + \alpha_s(\mu_R)\left(\frac{c_1^{(1)}(n,\nu,|\vec{k}_{J_1}|, x_{J_1})}{c_1(n,\nu,|\vec{k}_{J_1}|, x_{J_1})} + \frac{c_2^{(1)}(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}{c_2(n,\nu,|\vec{k}_{J_2}|, x_{J_2})}\right)\right.$$
$$\left. + \bar{\alpha}_s^2(\mu_R) \ln\left(\frac{x_{J_1} x_{J_2} s}{s_0}\right)\left(\bar{\chi}(n,\nu) + \frac{\beta_0}{8C_A}\chi(n,\nu)\left(-\chi(n,\nu) + \frac{10}{3} + \ln\frac{\mu_R^4}{\vec{k}_{J_1}^2 \vec{k}_{J_2}^2}\right)\right)\right],$$

where

$$c_1(n,\nu,|\vec{k}|, x_{J_1}) = 2\sqrt{\frac{C_F}{C_A}}(\vec{k}^2)^{i\nu-\frac{1}{2}}\left(\frac{C_A}{C_F} f_g(x,\mu_F) + \sum_{a=q,\bar{q}} f_q(x,\mu_F)\right), \tag{3.39}$$

$$c_2(n,\nu,|\vec{k}|, x) = \left[c_1(n,\nu,|\vec{k}|, x)\right]^*, \tag{3.40}$$

$$c_1^{(1)}(n,\nu,|\vec{k}|,x) = \frac{1}{\pi}\sqrt{\frac{C_F}{C_A}}(\vec{k}^2)^{i\nu-\frac{1}{2}}\int_x^1 \frac{d\zeta}{\zeta}\zeta^{-\bar{\alpha}_s(\mu_R)\chi(n,\nu)}\left\{\sum_{a=q,\bar{q}} f_a\left(\frac{x}{\zeta}\right)\right. \tag{3.41}$$

$$\times\left[\left\{P_{qq}(\zeta)+\frac{C_A}{C_F}P_{gq}(\zeta)\right\}\ln\frac{\vec{k}^2}{\mu_F^2} - 2\zeta^{-2\gamma}\ln R\{P_{qq}(\zeta)+P_{gq}(\zeta)\}\right.$$

$$-\frac{\beta_0}{2}\ln\frac{\vec{k}^2}{\mu_R^2}\delta(1-\zeta) + C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^2}+\frac{85}{18}+\frac{\pi^2}{2}\right.$$

$$\left.+\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\}$$

$$+(1+\zeta^2)\left\{C_A\left(\frac{(1+\zeta^{-2\gamma})\chi(n,\gamma)}{2(1-\zeta)_+}-\zeta^{-2\gamma}\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right)\right.$$

$$\left.+\left(C_F-\frac{C_A}{2}\right)\left[\frac{\bar{\zeta}}{\zeta^2}I_2-\frac{2\ln\zeta}{1-\zeta}+2\left(\frac{\ln(1-\zeta)}{1-\zeta}\right)_+\right]\right\}$$

$$+\delta(1-\zeta)\left(C_F\left(3\ln 2-\frac{\pi^2}{3}-\frac{9}{2}\right)-\frac{5n_f}{9}\right)$$

$$\left.+C_A\zeta+C_F\bar{\zeta}+\frac{1+\bar{\zeta}^2}{\zeta}\left\{C_A\frac{\bar{\zeta}}{\zeta}I_1+2C_A\ln\frac{\bar{\zeta}}{\zeta}+C_F\zeta^{-2\gamma}(\chi(n,\gamma)-2\ln\bar{\zeta})\right\}\right]$$

$$+f_g\left(\frac{x}{\zeta}\right)\frac{C_A}{C_F}$$

$$\times\left\{\left\{P_{gg}(\zeta)+2n_f\frac{C_F}{C_A}P_{qg}(\zeta)\right\}\ln\frac{\vec{k}^2}{\mu_F^2}-2\zeta^{-2\gamma}\ln R\{P_{gg}(\zeta)+2n_fPqg(\zeta)\}\right.$$

$$-\frac{\beta_0}{2}\ln\frac{\vec{k}^2}{4\mu_R^2}\delta(1-\zeta)+C_A\delta(1-\zeta)\left\{\chi(n,\gamma)\ln\frac{s_0}{\vec{k}^2}+\frac{1}{12}+\frac{\pi^2}{6}\right.$$

$$\left.+\frac{1}{2}\left(\psi'\left(1+\gamma+\frac{n}{2}\right)-\psi'\left(\frac{n}{2}-\gamma\right)-\chi^2(n,\gamma)\right)\right\}$$

$$+2C_A(1-\zeta^{-2\gamma})\left(\left(\frac{1}{\zeta}-2+\zeta\bar{\zeta}\right)\ln\bar{\zeta}+\frac{\ln(1-\zeta)}{1-\zeta}\right)$$

$$+C_A\left[\frac{1}{\zeta}+\frac{1}{(1-\zeta)_+}-2+\zeta\bar{\zeta}\right]\left((1+\zeta^{-2\gamma})\chi(n,\gamma)-2\ln\zeta+\frac{\bar{\zeta}^2}{\zeta^2}I_2\right)$$

$$\left.\left.+n_f\left[2\zeta\bar{\zeta}\frac{C_F}{C_A}+(\zeta^2+\bar{\zeta}^2)\left(\frac{C_F}{C_A}\chi(n,\gamma)+\frac{\bar{\zeta}}{\zeta}I_3\right)-\frac{1}{12}\delta(1-\zeta)\right]\right\}\right\}$$

$$c_2^{(1)}(n,\nu,|\vec{k}|,x) = \left[c_1^{(1)}(n,\nu,|\vec{k}|,x)\right]^* . \tag{3.42}$$

Here $\bar{\zeta}=1-\zeta, \gamma=i\nu-1/2, P_{i,j}(\zeta)$ are leading order DGLAP kernels. For the

$I_{1,2,3}$ functions we have results:

$$I_2 = \frac{\zeta^2}{\bar{\zeta}^2}\Bigg[\zeta\left(\frac{{}_2F_1(1, 1+\gamma-\frac{n}{2}, 2+\gamma-\frac{n}{2}, \zeta)}{\frac{n}{2}-\gamma-1} - \frac{{}_2F_1(1, 1+\gamma+\frac{n}{2}, 2+\gamma+\frac{n}{2}, \zeta)}{\frac{n}{2}+\gamma+1}\right)$$

$$+\zeta^{-2\gamma}\left(\frac{{}_2F_1(1, -\gamma-\frac{n}{2}, 1-\gamma-\frac{n}{2}, \zeta)}{\frac{n}{2}+\gamma} - \frac{{}_2F_1(1, -\gamma+\frac{n}{2}, 1-\gamma+\frac{n}{2}, \zeta)}{\frac{n}{2}-\gamma}\right)$$

$$+\left(1+\zeta^{-2\gamma}\right)\left(\chi(n,\gamma) - 2\ln\bar{\zeta}\right) + 2\ln\zeta\Bigg] \tag{3.43}$$

$$I_1 = \frac{\bar{\zeta}}{2\zeta} I_2 + \frac{\zeta}{\bar{\zeta}}\left[\ln\zeta + \frac{1-\zeta^{-2\gamma}}{2}\left(\chi(n,\gamma) - 2\ln\bar{\zeta}\right)\right], \tag{3.44}$$

$$I_3 = \frac{\bar{\zeta}}{2\zeta} I_2 - \frac{\zeta}{\bar{\zeta}}\left[\ln\zeta + \frac{1-\zeta^{-2\gamma}}{2}\left(\chi(n,\gamma) - 2\ln\bar{\zeta}\right)\right] . \tag{3.45}$$

Considering hadron-hadron scattering in the common parton model for two jet production at LO one deals with a back-to-back reaction and expects the azimuthal angles of the two jets always to be π and hence completely correlated. Increasing the rapidity difference between these jets, the phase space allows more emission leading to a decorrelation between the jets. If the rapidity differences the resummation leads to a description with BFKL theory. Leading logarithmic approximation overestimates this decorrelation.

In view of the investigation of the azimuthal decorrelation, it is useful to define:

$$\mathcal{C}_0 = \int d\phi_{J_1} d\phi_{J_2} d\sigma \ , \tag{3.46}$$

the coefficients $\mathcal{C}_n$ and the moments of the azimuthal decorrelations, which are defined as

$$\langle\cos(n\phi)\rangle = \frac{\int d\phi_{J_1} d\phi_{J_2} \cos[n(\phi_{J_1} - \phi_{J_2} - \pi)] d\sigma}{\int d\phi_{J_1} d\phi_{J_2} d\sigma} = \frac{\mathcal{C}_n}{\mathcal{C}_0} \ . \tag{3.47}$$

In the remainder of the paper the authors consider the study of the dependence of the $\mathcal{C}_n$ on Y in the center-of-mass energy $\sqrt{s}$ at LHC reference values. The value of cone size is $R = 0.5$.

To take into account the terms in $\mathcal{C}_n$ depending on the scales μ_R and s_0 (subleading in NLA) the authors adopted an adaptation of the *principle of minimal sensitivity* (PMS). According to this principle the optimal choices for μ_R and s_0 are those values for which the physical observables under examination express the minimal sensitivity to changes of both of these scales. This because the complete resummation of the perturbative series would not depend on the scale μ_R and s_0.

Among the conclusions of the authors there are the following:

- NLA corrections to the impact factors are very important and not be ignored in a consistent NLA BFKL analysis
- small-cone approximation is a valid approximation because for $R \to 0$ the dependence of the cross section on the jet cone parameter is

$$d\sigma \sim A \ln R + B + \mathcal{O}(R^2) \ . \tag{3.48}$$

  The pieces of order $\mathcal{O}(R^2)$ are presumably less important than the uncertainties related with the choice of the scales μ_R and s_0;
- PMS optimization procedure is expected to be a valid tool because it gives a larger values of energy factorization scale than the scale given by $\mu_R^2 = |\vec{k}_{J_1}||\vec{k}_{J_2}|$, and this is attributable to the presence of important contributions subleading to the NLA, especially for higher value of Y;
- For a kinematic scale $\mu_R^2 = s_0 = |\vec{k}_{J_1}||\vec{k}_{J_2}|$ and for high value of Y the prediction are not acceptable. This is because NLO correction of jet vertices are negative and very larger in absolute value. This is an indication of the need to use PMS procedure, in this case, to obtain plausible prediction;
- PMS approach is not working well in case of asymmetric kinematics.

3.3 NLA BFKL calculation with jet cone size treated exactly

Another approach to obtain a complete Mueller-Navelet jets treatment in next-to-leading BFKL theory is given in [9]. In this paper the authors calculated cross section and azimuthal decorrelation using jet cone algorithm in (2.19). The key points in this approach are the following:

- Energy scale s_0 viewed as a product of two energy scales

$$s_0 = \sqrt{s_{0,1} s_{0,2}} \ . \tag{3.49}$$

  Furthermore respect to that used in [7, 8]

$$s_{0,1} = (|\mathbf{k}_{J,1}| + |\mathbf{k}_{J,1} - \mathbf{k}_1|)^2 \to s'_{0,1} = \frac{x_1^2}{x_{J,1}^2 \mathbf{k}_{J,1}^2} \ , \tag{3.50}$$

$$s_{0,2} = (|\mathbf{k}_{J,}| + |\mathbf{k}_{J,2} - \mathbf{k}_2|)^2 \to s'_{0,2} = \frac{x_2^2}{x_{J,2}^2 \mathbf{k}_{J,2}^2} \ , \tag{3.51}$$

$$\frac{\hat{s}}{s_0} \to \frac{\hat{s}}{s'_0} = \frac{x_{J,1} x_{J,2} s}{|\mathbf{k}_{J,1}| \cdot |\mathbf{k}_{J,2}|} = e^{y_{J,1} - y_{J,2}} \equiv e^Y \ , \tag{3.52}$$

  This change in Green's function has to be accompanied by following correction term to the impact factor:

$$\Phi_{\mathrm{NLL}}(\mathbf{k}_i; s'_{0,i}) = \Phi_{\mathrm{NLL}}(\mathbf{k}_i; s_{0,i}) + \int d^2\mathbf{k}' \Phi_{\mathrm{LL}}(\mathbf{k}'_i) \mathcal{K}_{\mathrm{LL}}(\mathbf{k}'_i; \mathbf{k}_i) \frac{1}{2} \ln \frac{s'_{0,i}}{s_{0,i}} \tag{3.53}$$

  and the final form used is

$$V^{(1)}_{q;\,\mathrm{LL\ subtraction}} = -\frac{C_A}{\pi^2} \frac{1}{z(\mathbf{k}-\mathbf{k}')^2} \frac{(\mathbf{k}-\mathbf{k}')(\mathbf{k}-\mathbf{k}'-z\mathbf{k}')}{(\mathbf{k}-\mathbf{k}')^2 (\mathbf{k}-\mathbf{k}'-z\mathbf{k}')^2} V^{(0)}_q(\mathbf{k}', x) \ , \tag{3.54}$$

$$V^{(1)}_{g;\,\mathrm{LL\ subtraction}} = -\frac{C_A}{\pi^2} \frac{1}{z(\mathbf{k}-\mathbf{k}')^2} \frac{(\mathbf{k}-\mathbf{k}')(\mathbf{k}-\mathbf{k}'-z\mathbf{k}')}{(\mathbf{k}-\mathbf{k}')^2 (\mathbf{k}-\mathbf{k}'-z\mathbf{k}')^2} V^{(0)}_g(\mathbf{k}', x) \ ; \tag{3.55}$$

- As found with SCA method the effect of NLL corrections to the jet vertex function is very important, of the same order as the one obtained when passing from LL to NLL Green's function;

- The uncertainty due to change in μ_R, s_0 is drastically reduced when the NLL to the vertices are taken into account but is still sizeable;
- It seems that azimuthal decorrelation is almost not enhanced by an increasing rapidity.

Chapter 4

Comparison of predictions with CMS data

Recently it has become possible to compare theoretical analysis of Mueller-Navelet jet production process with first CMS data [41] at center-of-mass energy of 7 TeV. A first comparison with these experimental data according to the approach of jet cone algorithm of (2.19) does not lead to agreement with experiment [42]. Here the authors used a RG-improved kernel. This means that the NLA BFKL kernel is improved by imposing its compatibility with the DGLAP equation, although energy scales where not optimized. Later a similar analysis whose redone [43] using standard kernel, but energy scales optimized according to the *Brodsky-LePage-McKenzie method* (BLM). This method is a way of absorbing the non-conformal terms of the perturbative series in a redefinition of the coupling constant. To improve the convergence of the perturbative series, one should extract the β_0-dependent part of the observable and chose the renormalization scale to make it vanish. The authors found a nice agreement at the larger values of relative separation of jets Y. They define

$$\mathcal{C}_0 = \frac{d\sigma}{d|\vec{k}_{J,1}|d|\vec{k}_{J,2}|dy_{J,1}dy_{J,2}} \,, \tag{4.1}$$

and

$$\frac{\mathcal{C}_n}{\mathcal{C}_0} = \langle \cos(n(\phi_{J,1} - \phi_{J,2} - \pi)) \rangle \equiv \langle \cos(n\varphi) \rangle \tag{4.2}$$

analogously to Eqs. (3.34) and (3.35). The comparison with experimental data are shown in Figs. (4.1, 4.2, 4.3, 4.4) and (4.5). The two evaluations in figures are one with factorization scale $\mu_F{=}\mu_{R,\text{BLM}}$ and the other with "natural" $\mu_F{=}\sqrt{|\vec{k}_{J,1}|\cdot|\vec{k}_{J,2}|}$. The results first shown are those for the angular correlations $\langle\cos\varphi\rangle$, $\langle\cos 2\varphi\rangle$ and $\langle\cos 3\varphi\rangle$ as a function of relative rapidity Y (Figs. (4.1, 4.2) and (4.3), respectively).

The conclusion for these three observables is similar: when one uses the "natural" scale μ_F, the NLA BFKL calculation is always above the data. Data are much better described when setting the scale according to the BLM procedure. Instead, the ratios $\langle\cos 2\varphi\rangle/\langle\cos\varphi\rangle$ and $\langle\cos 3\varphi\rangle/\langle\cos 2\varphi\rangle$ are almost not affected by BLM procedure, see Figs. (4.4) and (4.5). This is because these observables are very stable with respect to the scales. The same observables are calculated in [44], where the adopted jet vertices are calculated in the approximation of small aperture of the jet cone. To improve the stability of perturbative series several methods have been devised for the optimal choice of the energy scales, including the BLM method. These new BLM calculations support the statement that theoretical predictions are in a rather good agreement with CMS data. The results are obtained in two variants of the BLM method, dubbed (a) and (b) in Figs. (4.6, 4.7, 4.9, 4.8) and (4.10) for the same observables (coefficients C_n are related to $\mathcal{C}_n$ of Eqs. (3.34), (3.35), in particular the ratios $\mathcal{C}_n/\mathcal{C}_0$ are the azimuthal correlations $\langle\cos n\phi\rangle$). For the two BLM methods (a) and (b) refer to a separate publication for details [45]. Nevertheless the prediction lies somewhat beyond the range of the theoretical uncertainty bound of [43]. This difference is related with a "representation uncertainty", related to the possiblity of contructing several different, but equivalent representations of the NLA BFKL amplitude.

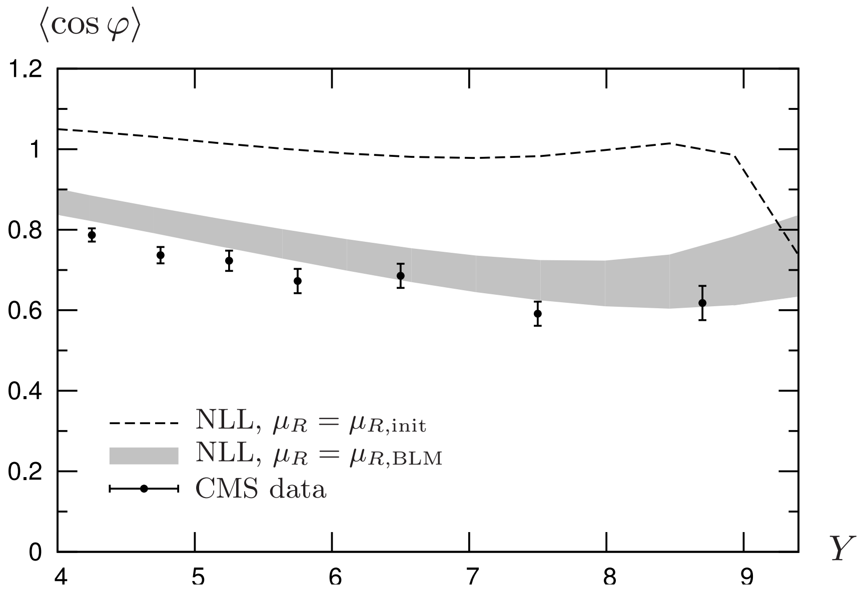

Figure 4.1: Variation of $\langle \cos \varphi \rangle$ as a function of Y at NLL accuracy compared with CMS data.

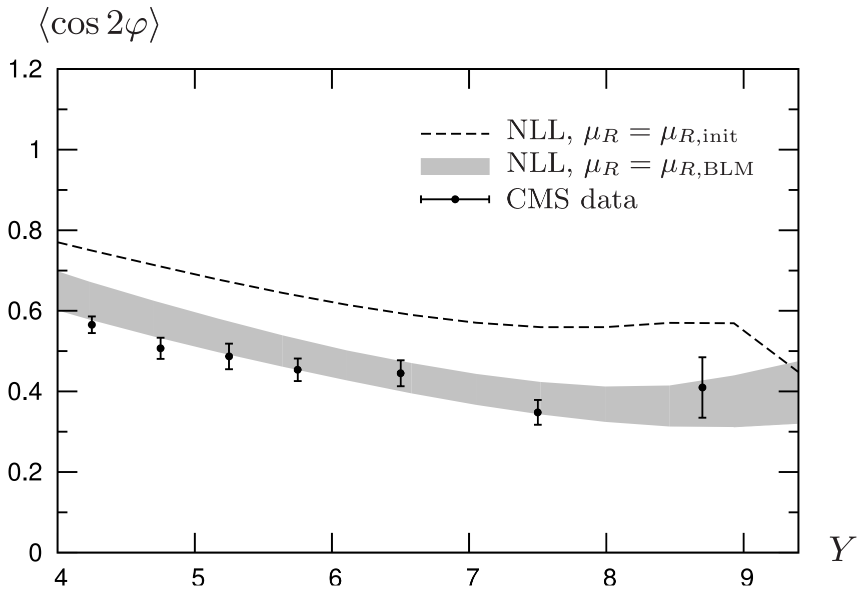

Figure 4.2: Variation of $\langle \cos 2\varphi \rangle$ as a function of Y at NLL accuracy compared with CMS data.

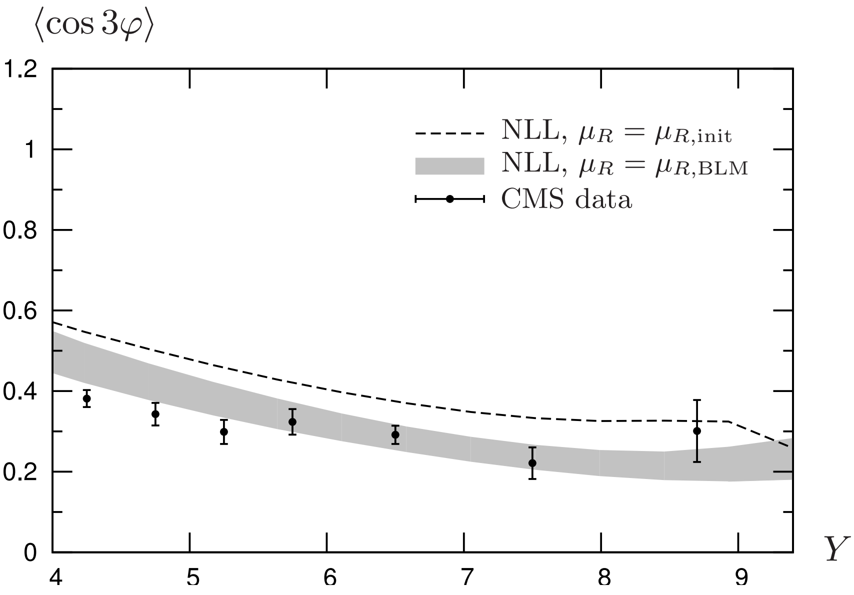

Figure 4.3: Variation of $\langle \cos 3\varphi \rangle$ as a function of Y at NLL accuracy compared with CMS data.

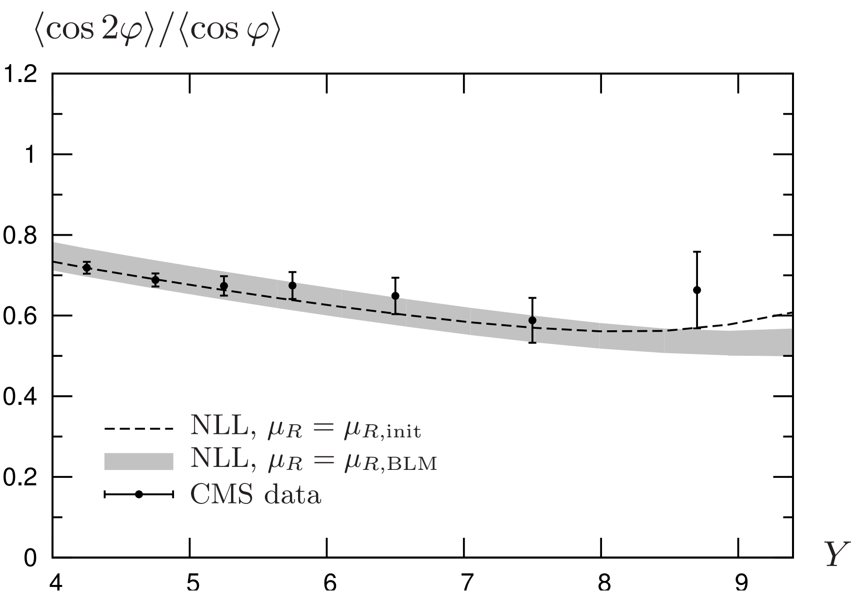

Figure 4.4: Variation of $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ as a function of Y at NLL accuracy compared with CMS data.

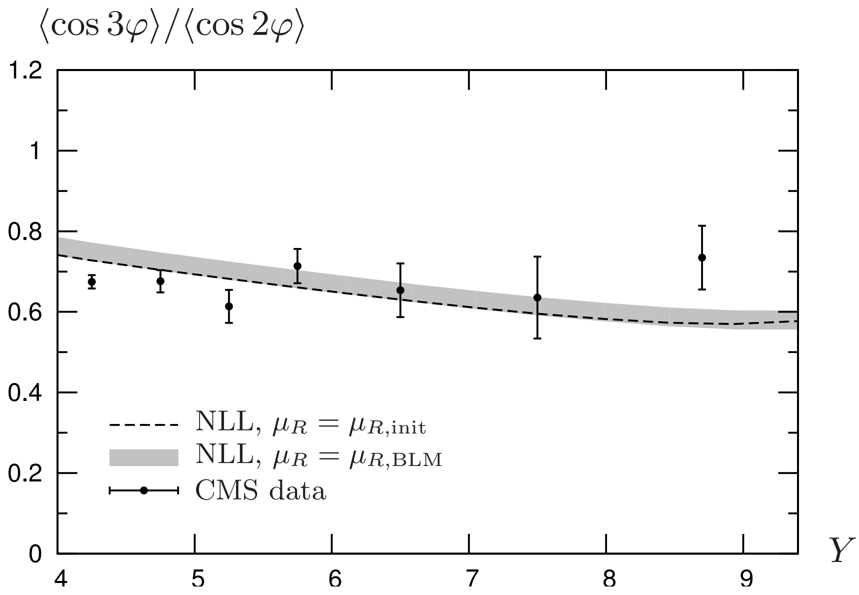

Figure 4.5: Variation of $\langle \cos 3\varphi \rangle / \langle \cos 2\varphi \rangle$ as a function of Y at NLL accuracy compared with CMS data.

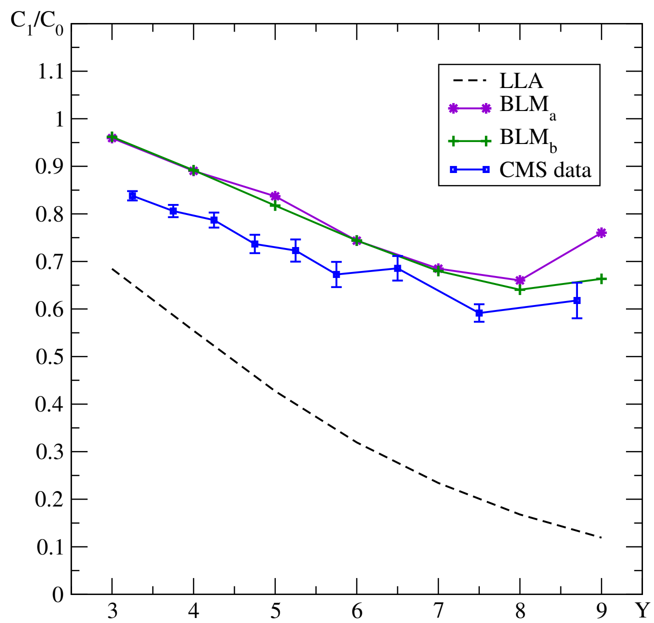

Figure 4.6: Y dependence of azimuthal correlations C_1/C_0.

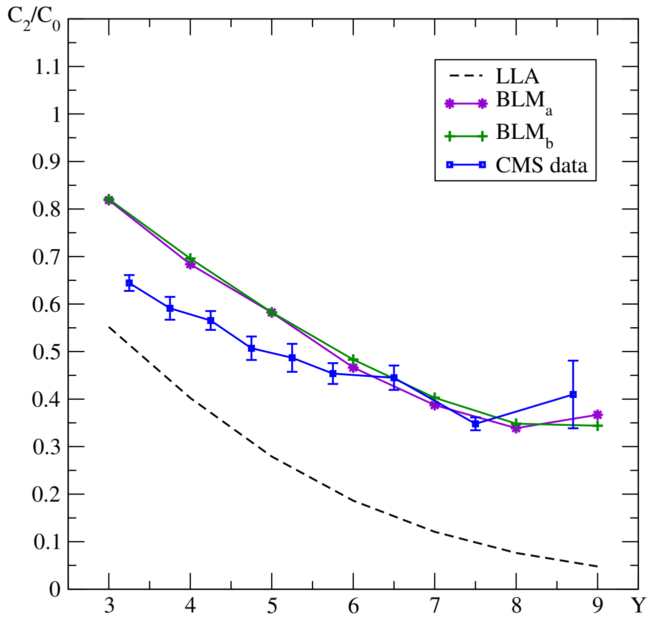

Figure 4.7: Y dependence of azimuthal correlations C_2/C_0.

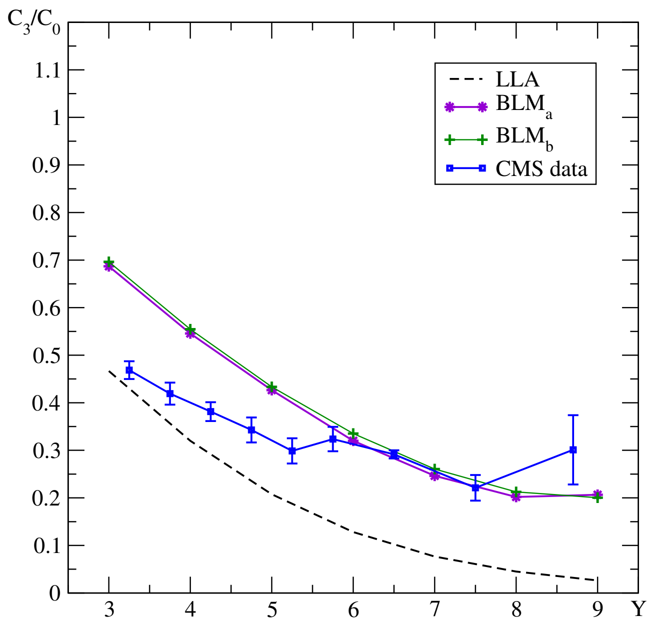

Figure 4.8: Y dependence of azimuthal correlations C_3/C_0.

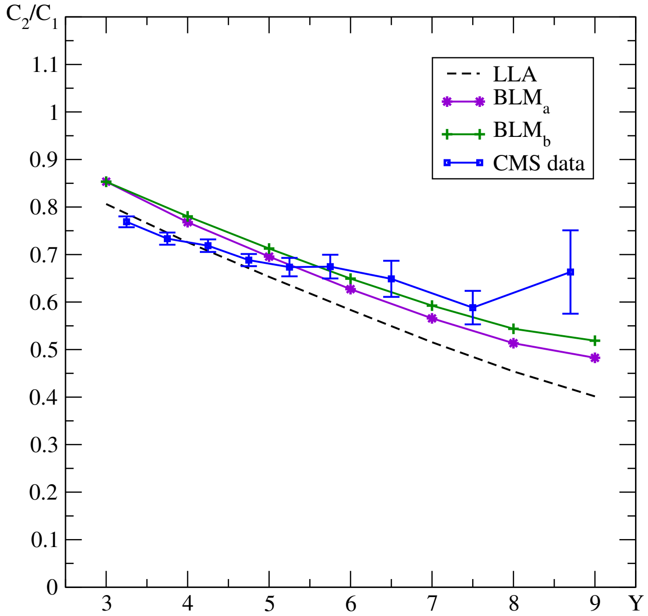

Figure 4.9: Y dependence of azimuthal correlations ratios C_2/C_1.

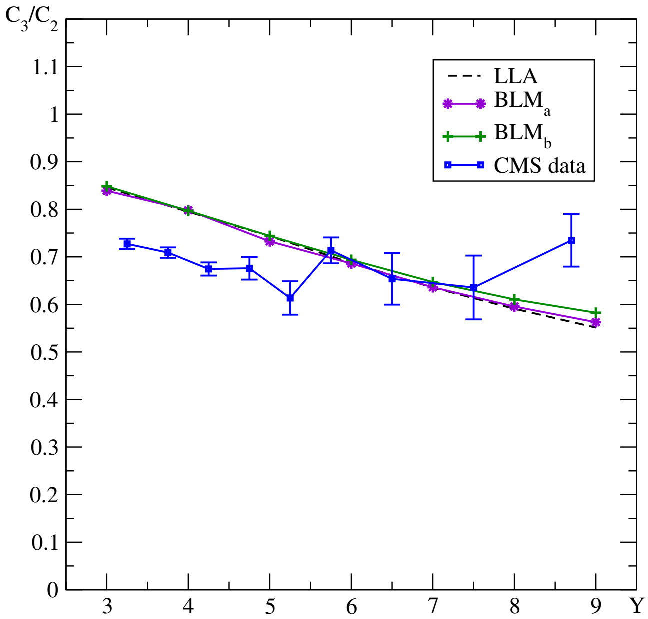

Figure 4.10: Y dependence of azimuthal correlations ratios C_3/C_2.

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