Abstract

In the present thesis the macroscopic non-linear behavior of composite materials with a periodic and heterogeneous microstructure is studied. There are many different kinds of phenomena that produce non-linear effects in composite materials, for example intralaminar damage, delamination and microbucking in fiber reinforced composite or micro-cracking in cellular materials. In this work attention is devoted to the mechanical modeling of nonlinear phenomena associated to the presence of micro-cracks in the context of linear elasticity and of microscopic instabilities in the framework of the finite strain theory. Applications have been developed with reference to microstructures of cellular type and with embedded inclusions. The thesis is structured according to the following chapters:

-In the first chapter the fundamental concepts of the finite strains theory are recalled. The constitutive relations associated to a class of conjugate stress-strain pairs are introduced. The basic expressions of the incremental constitutive laws are shown with special reference to incrementally linear constitutive laws. Finally the stability and the uniqueness of the equilibrium solution are analyzed.

-In the second chapter, after an introduction about the homogenization techniques, the micro and macro stability phenomena occurring in composite materials with a periodic microstructure are studied from a theoretical point of view in the context of the finite strains theory. The formulation starts from a variational formulation of the problem. Novel macroscopic measures of micro-structural stability are introduced corresponding to the positive definiteness of the homogenized moduli tensors relative to a class of conjugate stress-strain pairs and their effectiveness to obtain a conservative prediction of the microscopic primary instability load is pointed out.

Analysis of these stability phenomena plays a fundamental role because often the collapse of composite materials with periodic microstructure is related to microstructural instabilities. In addition the microscopic stability analysis establishes the region of validity of the standard homogenization procedure based on the unit cell procedure.

-In the third chapter, in the context of the small strains theory, non-linear phenomena are presented with reference to composite materials with a porous microstructure containing micro-cracks spreading from the voids. The fundamental techniques of homogenization are applied in conjunction with fracture mechanics theory and interface models. The energy release rate is evaluated through the *J*-integral technique.

-In the fourth chapter some numerical applications carried out by means of a one-way coupled finite element code, are proposed.

In the first section the numerical results will be introduced with reference to the theoretical aspects developed in the second chapter. Numerical analyses are addressed to composite materials with a periodic microstructure, namely a porous microstructure and a particle-reinforced microstructure. The adopted constitutive law is hyperelastic. Periodic boundary conditions will be used for the microstructure, and uniaxial and equibiaxial loading conditions are considered. Numerical analyses are able to show the exact region of microscopic stability, obtained by taking into account all the micro-structural details, and the region of macroscopic stability, determinate by studying homogenized material properties.

To elaborate macroscopic criteria able to give a conservative prediction of the microstructural stability, different measures of macroscopic instability are introduced with reference to work conjugate strain-stress measures.

In the second section of this chapter a numerical analyses with reference to the micromechanical model proposed in the third chapter is developed. In this case the microstructure adopted for the composite materials is a cellular microstructure in which there is the presence of two micro-cracks advancing symmetrically from the void. The microstructure is subjected to three different boundary conditions namely respectively: linear displacements, periodic fluctuations and antiperiodic tractions and uniform tractions.

The objective of this section is to verify the validity of the homogenization technique in the prediction of micro-crack evolution phenomena, for composites with locally periodic microstructure.

The energy release rate obtained through the micromechanical model will be compared with a 2D composite structure composed by a regular arrangement of 5x5 unit cells. The composite structure is subjected to two different boundary conditions: the former is associated with the absence of contact between the surfaces of the micro-cracks, on the contrary in the latter case there is the presence of the contact. This type of comparison allows to investigate the accuracy of the proposed procedure in presence of macroscopic tension and strain gradients.